feature article

Magnetic dynamos in the lab

Daniel P. Lathrop and Cary B. Forest

There is as yet no predictive theory of planetary or astrophysical dynamos. But theorists, numerical modelers, and experimenters are on the case.

Daniel Lathrop is a professor of physics and geology at the University of Maryland, College Park. **Cary Forest** is a professor of physics at the University of Wisconsin–Madison.

All astrophysical plasmas are, as far as we know, magnetized and turbulent. They range in size, density, and temperature from relatively small, dense stellar plasmas to enormous, diffuse plasmas in clusters of galaxies. The magnetic fields and the turbulence play important roles in issues as diverse as cosmological structures formation and the origin of cosmic rays.

Closer to home, Earth also has a magnetic field, as do most but not all of the other planets. The geomagnetic field makes our planet more hospitable by shielding us from the Sun's charged-particle wind. We have only a partial understanding of how the turbulent flow of liquid iron in Earth's outer core generates the geomagnetic field. (Despite being hotter, the inner iron core is solid because it's under greater pressure.)

That level of understanding does not yet make predictive forecasting possible, which is mildly alarming because the geomagnetic field has fallen monotonically throughout recorded history—by approximately 10% since 1838, when Carl Friedrich Gauss published his pioneering global models. We may be headed for a magnetic reversal. The geomagnetic field has reversed many hundreds of times over geological history; the last one happened 780 000 years ago. We can't be sure about changes in solar radiation during a reversal, when the field is bound to be weak.

The origins and dynamics of the dynamo-generated magnetic fields of Earth, the Sun, the gas-giant planets, and nearly every massive astrophysical object are almost certainly controlled by complex turbulent flows of plasmas or conducting liquids. The quest to understand the dynamo process comprises theoretical, computational, and experimental undertakings. Here are some of the outstanding questions:

▶ Why do some planets and stars have strong surface magnetic fields and others do not, and what sets those strengths?
▶ How do stars and galaxies develop large-scale magnetic fields?
▶ What is the role of rotation in the dynamo process?
▶ What determines time-varying behaviors such as reversals of Earth's field and oscillations of the Sun's field?
▶ What are the roles of dynamos and magnetic fields in protoplanetary, protostellar, and accretion disks?

The issues span many disciplines. Geophysics, astrophysics, plasma physics, and planetary science all claim the dynamo problem as their own. Why, beyond scientific curiosity, do we want a predictive dynamo science? Because we worry about solar storms and the Sun's role in global climate change. And on a longer time scale, there's the declining geomagnetic field.

From the perspective of two experimentalists, this article attempts to describe the current experimental, theoretical, and computational state of the field. Our view is that true understanding pivots on having a theory that yields predictions testable in the laboratory and extrapolatable to geophysical and astrophysical settings.

What is a dynamo?

The basic dynamo process is a mechanism that converts kinetic energy to magnetic energy, much as an electric generator does. The underlying physics is Faraday induction, whereby motion couples to electromagnetic fields (see the box on page 41).

Fundamentally, dynamos exist when a moving conductor serves to amplify a seed magnetic field. The field of a natural dynamo with a fluid conductor evolves as a feedback-driven instability, until the Lorentz forces begin to modify the underlying velocities and bring about saturation—the cessation of field growth.

The three basic principles that govern dynamo action are Ampère's law (the creation of magnetic fields by electric currents), Faraday's law (the creation of electromotive forces by time-varying magnetic fields), and Ohm's law for a moving conductor (the creation of currents by the Lorentz force). Of course, special relativity tells us that the last two are intimately related.

The relative importance of magnetic-field generation by fluid flow and resistive decay of the field is quantified by the dimensionless magnetic Reynolds number $Rm = UL/\eta$, where U and L are the system's characteristic velocity and length scales and η is the fluid's magnetic diffusivity, which is proportional to its resistivity. When Rm is very large, magnetic fields can be viewed as frozen into the moving fluid. And when Rm is small, magnetic fields rapidly dissipate. The more familiar hydrodynamic Reynolds number Re, which governs the appearance of turbulence in fluids, is given by UL/ν, where ν is the fluid's kinematic viscosity.

Natural dynamos exploit a feedback loop: A seed magnetic field can generate currents through motional electromotive forces; those currents can then reinforce the magnetic field. The conductor's velocity field is the energy source of gain, and electrical resistance dissipates magnetic energy to heat.

The cartoon panels in the box illustrate the basic phe-

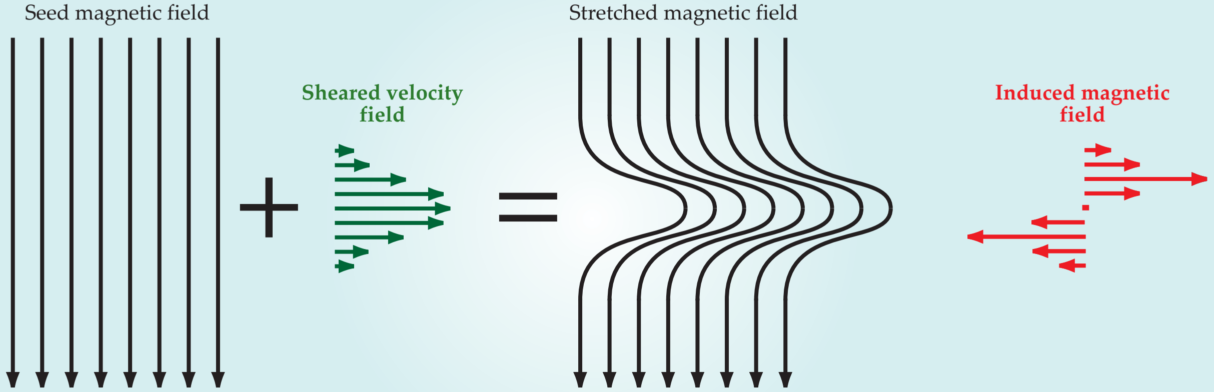
Dynamo tutorial

A dynamo is a magnetic field generated by motions of a conducting medium. All naturally occurring dynamos share several key ingredients. The medium is a fluid—plasma or liquid metal—sufficiently conducting that electrical resistance does not dominate over the coupling of its motion to the magnetic field. For a good conductor, the medium's flow drags and stretches magnetic field lines via Faraday induction.

One sort of stretching happens in sheared velocity fields (see step 1 below). Bending of the magnetic field induces more magnetic field. But emergence of a self-generating dynamo requires an additional mechanism to convert some of the induced field back into a pattern shared by the initial magnetic field. To that end, the medium's motion must bend or twist the field (see step 2 below). That second step adds energy to the initial magnetic field and thus reinforces it.

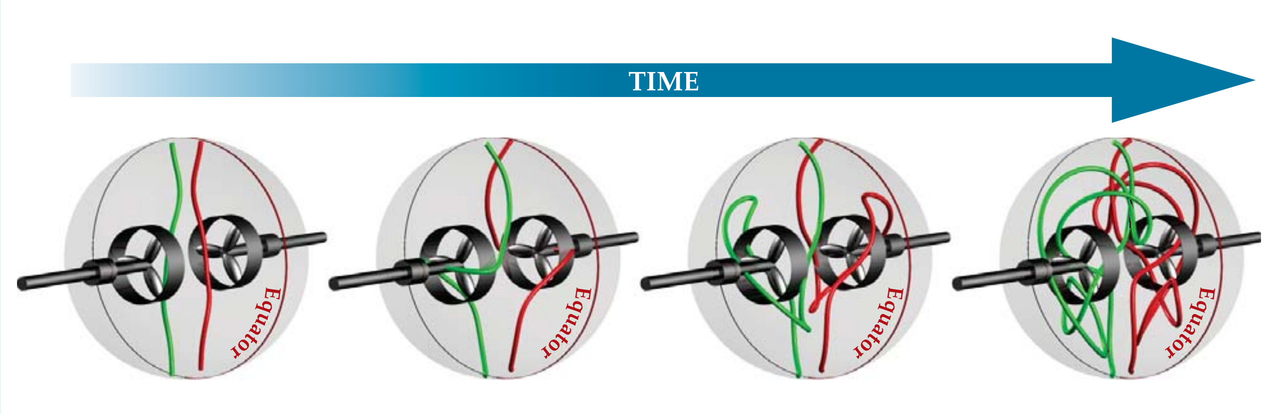
Such a dynamo process obviously needs an initial seed field. But if the gain mechanism described above is available, adequate seeds can easily come from external or thermal sources. And after the magnetic field has gained sufficient strength, its nonlinear feedback on the medium's flow leads to saturation in a self-organized steady state.

Step 1: Shear flow induces new field.



When the medium's conductivity is large, its velocity field stretches magnetic-field lines via induction as if the field were frozen into the moving fluid. That effect can be thought of as a consequence of Lenz's law. The stretching generates a strong magnetic field transverse to the direction of the seed field, and the process continues until the field's magnetic tension is strong enough to slow the flow. But because the seed field is not yet reinforced, a dynamo has not yet been created. That requires the second step.

Step 2: Twisting closes the feedback loop.



This sequence of images illustrates how an initial seed magnetic field in a laboratory setting can be stretched, twisted, and folded to reinforce itself. Two counterrotating propellers spin inside a planetary-dynamo surrogate: a spherical vessel filled with conducting fluid. They cause the fluid to rotate in opposite directions in each hemisphere and thrust outward toward the poles. The flow has strong shear.

Red and green lines are flux lines evolving from seed field lines initially in opposite hemispheres. By the second panel, stretching of the seed field lines is amplifying field strength and increasing the magnetic energy. By the third panel, magnetic-field direction near the vessel's surface is opposite to that of the seed field near the center. By the last panel, the stretched field is reinforcing the central field and thus closing the feedback loop.

nomena with a simple flow field. But natural and laboratory flows leading to dynamos are almost always turbulent, with strong nonlinearities. Sometimes turbulence is helpful and necessary for creating fields. But it could also be detrimental and preclude the growth of a dynamo. The basic issue is not the lack of good physical models. The underlying equations are clear, but their solutions are complex and not yet fully understood. The central intellectual challenge is to describe the turbulent fluid flow, the resulting turbulent magnetic fields, and their interconnected dynamics.

The hydrodynamic Reynolds number is a dimensionless measure of the relative strength of flow nonlinearities and the smoothing effects of viscosity. A large *Re* brings turbulence. Figure 1 shows the parameter space spanned by *Re* and *Rm* for natural and laboratory dynamos. Lacking a predictive theory, planetary scientists confront the mystery of why some planets sustain dynamos and others don't. It's likely that all the planets in our solar system harbor liquid cores. But Mars has no active dynamo, although evidence points to its having had one in the past. Venus, about the same size as Earth and with a hotter interior, nonetheless has no dynamo. Is it because Venus is a slow rotator (with its day and year nearly the same) and seems to lack plate tectonics?

Another theoretical challenge is to understand how systems with extremely large *Rm*, such as the solar interior (10^8) and intragalactic plasmas (10^{15}), produce large-scale magnetic fields from small-scale turbulence. Models quite easily generate small-scale magnetic fields, but getting small fields to organize on large scales would require more complicated dynamics.

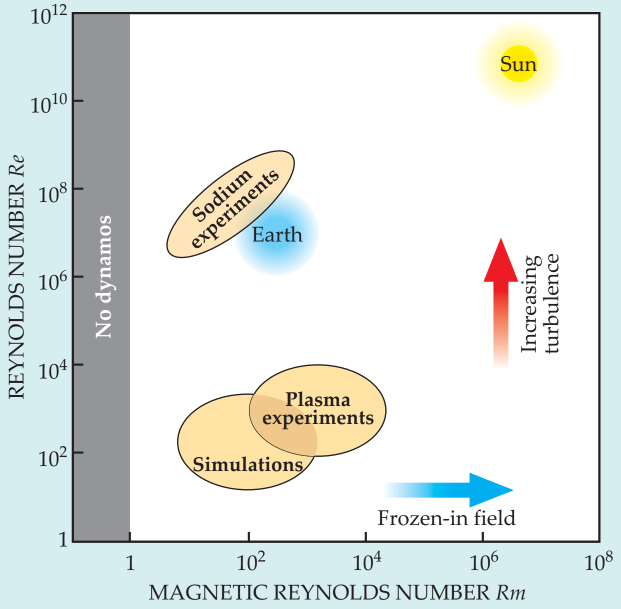


Figure 1. Magnetic dynamos with fluid conductors are characterized by two dimensionless parameters described in the text: the Reynolds number *Re*, which determines the turbulence of the fluid flow, and its magnetic analogue *Rm*. The figure shows regions of parameter space covered by familiar natural dynamos and regions accessible to computer simulations and laboratory experiments. With present-day computing power, simulations are difficult unless the two parameters are of the same order and neither one exceeds 10^3. Liquid-sodium experiments are useful for modeling planetary dynamos, but plasma experiments will probe much higher *Rm*. Theory precludes dynamo self-generation for *Rm* less than 1.

Numerical simulation

Direct numerical simulations of dynamos have yielded considerable progress on such theoretical puzzles. The present generation of laboratory dynamo experiments is, in no small way, motivated by advances in computing power. Kinematic dynamo calculations, in which the flow is specified and varied in search of growing magnetic eigenmodes, provided experimenters with examples of simple flows that have guided the laboratory models.[1,2]

Those early simulations have led to fully nonlinear dynamo simulations in which the evolution of the velocity field was carefully followed. Breakthrough geodynamo simulations by Gary Glatzmaier and Paul Roberts in 1995 showed that thermal convection could lead to a dynamo that exhibited Earth-like dipole magnetic structure and reversals.[3] Many groups have since explored the numerically accessible parameter space for convective dynamos.[4] The parameter space of such simulations is limited mostly by the computational ability to resolve turbulence on small scales.

But the very success of those simulations is puzzling. They were accomplished, of necessity, by assigning unrealistically high viscosities. So the numerical dynamos lack the realistic turbulence and wave modes believed to occur in the liquid-metal core of a real planet. In a planetary core, Coriolis, magnetic, and buoyant restoring forces should make the core a very wavy place.

Computational limitations on dealing simultaneously with the large range of dynamically active scales in stars force numerical simulations of the Sun to employ even greater effective viscosities. Nonetheless, the Sun's dynamo is in some ways better understood than Earth's. Turbulence in the convective zone below the solar surface governs the magnetic field and also drives large-scale flows that can be directly compared with high-fidelity reconstructions of the Sun's internal winds from helioseismology data. The Sun is very turbulent; it vigorously churns out small-scale magnetic fields that are readily seen in simulations.[5] Recent simulations have begun to capture the Sun's large-scale flows and magnetic fields.[6]

Closely related to the dynamo process in astrophysics is the magnetorotational instability that is thought to occur in galactic disks, stellar accretion disks, and protoplanetary nebulae. That mechanism can couple magnetic fields and shear flows to cause turbulence and magnetic-field growth in such astrophysical settings. Significant theoretical and numerical work has been done, for example, to understand the growth of magnetic fields via the instability of slow plasma waves in shear flows.[7] The magnetorotational instability may be important early in the history of stars and planets as a way of initiating a large seed field for dynamo mechanisms that require such a start.

Laboratory experiments

Experiments trying to mimic natural dynamos in the laboratory cannot directly match the Reynolds numbers of astrophysical objects. They can, however, reach *Rm* similar to Earth's and *Re* beyond what's possible in numerical simulations. To date, experiments investigating self-generation of magnetic fields have used liquid sodium as a conducting medium, but plasma experiments are coming soon.

Being the best electrical conductor of any liquid, sodium provides the maximum induction for any given flow. To reach the requisite *Rm* region for investigating magnetic-field generation, one needs large experimental volumes with high

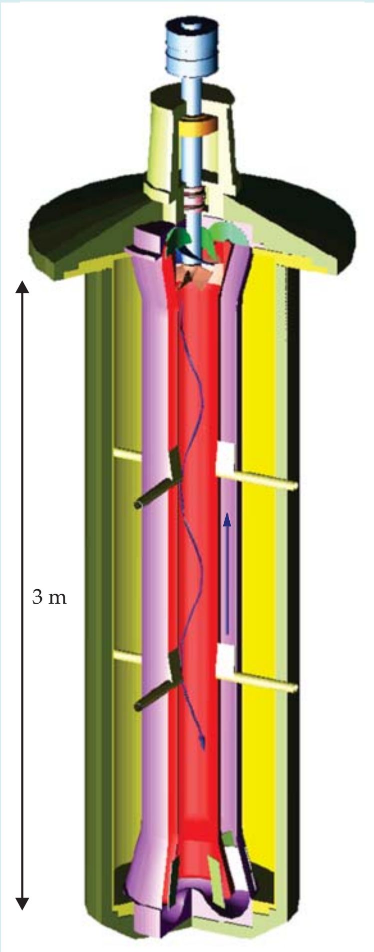


Figure 2. A pioneering experiment at the University of Latvia in Riga was one of two[9,10] that demonstrated dynamo self-generation in 2000 with liquid sodium constrained to execute helical flow. In the Riga device, shown here, a propeller and baffles made the sodium spiral downward in the vessel's central column (red) and then return straight upward in the cylindrical region (pink). The flow regions were surrounded by initially nonflowing liquid sodium (yellow). The constrained helical motion of the conducting fluid was a key component driving the dynamo. Turbulence was suppressed by the tightly constrained flow geometry. (Drawing courtesy of A. Gailitis.)

flow rates. The hazards of dealing with liquid sodium make the design and operation of such experiments challenging but manageable with present engineering practices.

Idealized liquid-metal flows generate magnetic fields when *Rm* exceeds some threshold, perhaps 100. A back-of-the-envelope estimate highlights the experimental challenge: The conductivity of sodium (which melts at a convenient temperature of 97 °C) decreases with increasing temperature, and its magnetic diffusivity η is about 0.1 m^2/s, six times that of copper. Obtaining a self-generating dynamo then requires the product *UL* to be at least 10 m^2/s. So to exceed threshold, a lab experiment with a typical size of 1 m would require fluid velocities of about 10 m/s. The power consumption of such a system would be on the order of 100 kW.

The example illustrates how laboratory experiments can reach dynamo conditions for planets, but not stellar or astrophysical regimes, where the threshold *Rm* is more like 1000, basically because the scaling of the required power is prohibitive. The power required to drive the flow scales as Rm^3. So a meter-sized laboratory reproduction of an astrophysical dynamo would require 100 MW!

Experimental investigations of dynamos have evolved from early ones in the 1960s that used rotating solid cylindrical conductors embedded in larger stationary conductors[8] to pioneering liquid-metal experiments in 2000 at the University of Latvia in Riga[9] and the University of Karlsruhe in Germany.[10] In the Karlsruhe and Riga experiments (see figure 2), pump-driven helical flows of liquid sodium through pipes were found to be suitable for generating magnetic fields. The desire to test helical motion was, in fact, an important motivation for using liquid metal.

The magnetic fields generated in the Riga and Karlsruhe experiments matched the predictions. The observed threshold *Rm* values were governed by the mean velocity fields, which implies that turbulence did not play a significant inhibiting role. Those observations contradicted the longstanding magnetohydrodynamic expectation that turbulence should greatly enhance the effective electrical resistivity of the flow.

That contradiction is now understood to result from the stark scale separation between the characteristic sizes of the magnetic field and the turbulence in those experiments. When turbulent eddies are much smaller than the system's overall scale, their damping effect on magnetic fields is minimized, and the mean flow (which ignores the eddies) governs the dynamo. More generally, unraveling the role of turbulence in enhancing dissipation remains a fundamental issue.

The most recent liquid-sodium experiment to self-excite a dynamo is the Von Karman Sodium device (VKS) at the French Atomic Energy Commission's laboratory in Cadarache. The Cadarache device, shown in figure 3a, has exhibited diverse and interesting dynamo action, including intermittent and steady dynamo states and chaotic Earth-like field reversals.[11] But unlike the earlier experiments, it generates a magnetic field—a dipole aligned with the rotation axis of the propeller that stirs the liquid—that can't be explained by theories invoking only the mean-flow states.

Subsequent analysis of Cadarache results suggests that the observations might be explained by differential fluid rotation combined with the generation of coherent small-scale vortices at the edges of the propeller blades and augmented by field amplification associated with the propeller's ferromagnetism. But the precise augmenting effects of the VKS's soft-iron propellers and of its somewhat restrictive flow-boundary conditions are not yet fully understood. Note that Earth's core is too hot to be ferromagnetic. The Cadarache experiment has raised many new questions that deserve further investigation.

Current experimental work is being done with more open, Earth-like vessel geometries at the University of Grenoble in France and at the universities of Wisconsin and Maryland. The Wisconsin device, shown in figure 3b, drives the flow by means of two propellers just inside opposite poles of its 1-m-diameter spherical vessel filled with liquid sodium (see PHYSICS TODAY, February 2006, page 13). The goal at Wisconsin is to achieve critical *Rm* for self-exciting dynamo action. But subcritical experiments in that apparatus have already discovered that system-size fluctuations generate currents that enhance the fluid's effective resistivity and revealed how turbulence can repress field generation.[12]

The Grenoble experiment, designed to resemble Earth's dynamo configuration even better, confines liquid sodium between a rotating outer sphere and a highly magnetized rotating inner sphere—meant to simulate, respectively, Earth's mantle and its solid-iron inner core. Shown in figure 3c, the experiment has revealed a wide range of wave modes and Lorentz-forced jets.[13] Liquid-sodium experiments at Maryland, with vessel diameters increasing from 30 to 60 cm, have shown turbulent induction, Coriolis-restored inertial waves, and magnetic instabilities related to the magnetorotational instability.[14] The turbulence, however, complicates the magnetorotational-instability interpretation of those observations. Related, astrophysically motivated liquid-gallium and liquid-sodium experiments are being

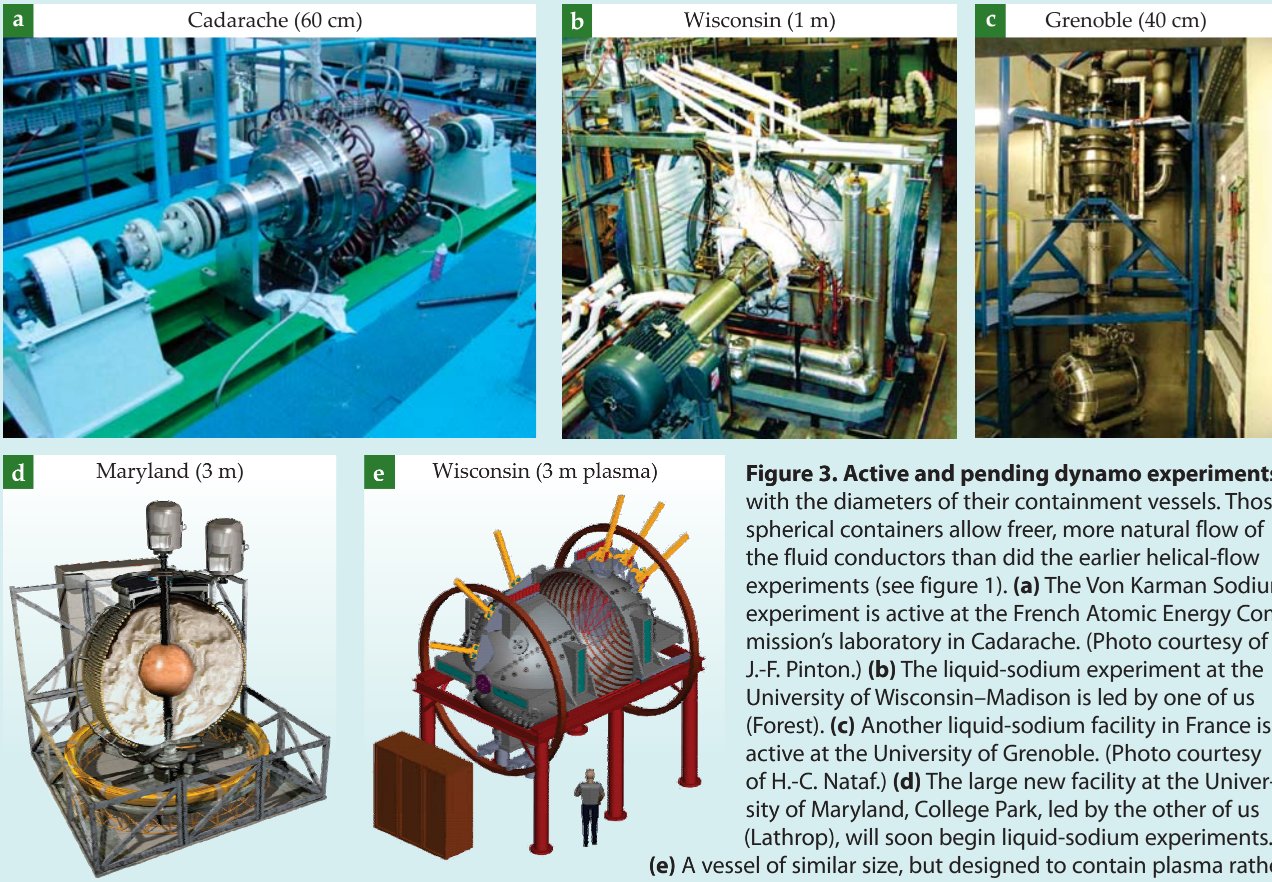


Figure 3. Active and pending dynamo experiments, with the diameters of their containment vessels. Those spherical containers allow freer, more natural flow of the fluid conductors than did the earlier helical-flow experiments (see figure 1). **(a)** The Von Karman Sodium experiment is active at the French Atomic Energy Commission's laboratory in Cadarache. (Photo courtesy of J.-F. Pinton.) **(b)** The liquid-sodium experiment at the University of Wisconsin–Madison is led by one of us (Forest). **(c)** Another liquid-sodium facility in France is active at the University of Grenoble. (Photo courtesy of H.-C. Nataf.) **(d)** The large new facility at the University of Maryland, College Park, led by the other of us (Lathrop), will soon begin liquid-sodium experiments. **(e)** A vessel of similar size, but designed to contain plasma rather than liquid metal, is under construction at Wisconsin.

done at Princeton University and the New Mexico Institute of Technology.

At Maryland, a 3-m-diameter experiment, shown in figure 3d, is scheduled to begin liquid-sodium operation this year. It should push the boundaries of experimentally accessible *Rm* values up to about 900, which is in the range that's estimated for Earth's core. Figure 3e shows a 3-m-diameter plasma-dynamo experiment under construction at Wisconsin. It should make possible a broader range of Reynolds parameters than is accessible to liquid-sodium experiments.

The successes of the Riga, Karlsruhe, and Cadarache experiments in achieving self-generating dynamo action owed much to their constrained helical flows. Therefore, an important motivation for the Grenoble, Wisconsin, and Maryland liquid-sodium efforts has been to do the same with more difficult, but more planetlike open geometries.

What we don't yet know

Despite all the experimental and theoretical efforts to date, significant unknowns remain. Without an adequate theory, we don't know the rate of the future decline of Earth's magnetic field and the scope of the magnetosphere's consequent contraction. And we have only limited ability to predict the solar sunspot cycle. There has been significant progress in understanding the strength of the dipole components generated in numerical attempts to match the observed geomagnetic field.[15] But there is as yet no predictive general theory for the strengths of planetary magnetic fields.

Most planets are rapid rotators. In the atmospheres of Earth and the Jovian planets, Coriolis effects manifest themselves clearly in jet streams and zonal winds. So fluid planetary cores must surely have rotation-dominated dynamics. None of the three liquid-metal experiments (Riga, Karlsruhe, and Cadarache) that have thus far demonstrated dynamo self-generation has had rotating container walls, so there is still much to be learned about dynamo regimes dominated by Coriolis forces.

Buoyancy and stratification are also surely important in Earth's core and the Sun's convective zone. Convection is presumably the power source for both the geo- and heliodynamos. The prospects for laboratory convective dynamos are hampered by the relatively weak velocity fields in laboratory convective flows. In the absence of gigantic devices well beyond the current generation, *Rm* will not exceed 1 in convection-driven experiments, thus precluding dynamo action.

With *Rm* exceeding 10^6, astrophysical dynamos are often coherent on scales much larger than their turbulence scales. The solar magnetic field has large-scale dipole and toroidal components, but the convection is dominated by scales much smaller than the Sun's radius. Galaxies have highly ordered fields on scales of thousands of light-years, typically a hundred times larger than the eddies of their supernova-driven turbulence. By contrast, in both numerical simulations and analytic calculations, turbulent dynamos with high *Rm* more readily generate magnetic fields whose coherence extends no farther than the turbulence scale.

Although *Rm* is the essential parameter governing magnetic-field generation in an experimental dynamo, *Re* is the most important parameter controlling the fluid's flow properties—aside from the experiment's geometry and its stirring mechanism. Large *Re* exceeding 10^4 is usually associated with strong turbulence, while low *Re* implies laminar flows with strong viscous dissipation. A limitation of liquid-sodium experiments has been the inability to vary the ratio $Pm \equiv Rm/Re$ (called the magnetic Prandtl number) and control the degree of turbulence. For liquid sodium and many other metals, *Pm* is about 10^{-5}. So liquid-metal flows, even with modest *Rm*, are always associated with very strong turbulence. If experimenters had a strongly conducting but viscous fluid for which *Rm* and *Re* were comparable, they would be able to compare the performance of turbulent and laminar dynamos.

The magnetohydrodynamics of incompressible, single fluids, though appropriate for describing liquid-metal experiments, does not exhibit the richness of processes that real astrophysical dynamos have. Astrophysical plasmas are compressible and heterogeneous, and they often encompass neutral particles that can affect their dynamics. At low plasma densities, electron and ion flows differ significantly once a magnetic field begins to grow, which gives rise to the Hall effect and interesting saturation dynamics. There may be new mechanisms for magnetic-field generation when the plasma is sufficiently collisionless that pressure can become anisotropic in a weak field.[16] And compressibility implies that stratified plasmas can exhibit buoyancy-driven instabilities.

How we might find out

The various challenges discussed above are being attacked with a combination of theoretical and experimental approaches. As more computational power becomes available, finer-scale simulations continue to push toward realistic levels of turbulence.

The next frontier in experimental dynamo studies is the push to higher values of *Rm*, ideally an order of magnitude beyond what's achievable in present liquid-sodium experiments. Steps in that direction are the initial liquid-sodium experiments slated for the Maryland 3-m device. That facility has until now been operating with water, to study the rotating hydrodynamic states that will also underlie the sodium flows at an *Rm* of about 900. Already the water experiments have established that Coriolis effects are dominant, as they are expected to be in planetary cores. The water experiments have also revealed rich precession-driven flows from torques actually due to Earth's diurnal rotation.

Plasmas are obvious candidates for pushing to higher *Rm*. In plasmas, conductivity increases significantly with increasing temperature, and laboratory plasmas can flow at tens of kilometers per second. For example, a plasma experiment in a 2-m-diameter vessel could achieve $Rm = 1000$ with a flow velocity of 5 km/s and an electron temperature of only 10 eV. And because a plasma's viscosity depends on its density and ion mass, experimenters could vary the viscosity at will. So they have, in effect, a knob to turn turbulence on and off. They might, for example, create noble-gas plasmas with *Re* ranging from 100 (with 10^{11} helium ions per cm^3) to 10 000 (with 10^{13} argon ions per cm^3), a variation that any liquid-metal experimenter would envy. A plasma experiment with *Re* much smaller than *Rm* would probe the regime of hot stellar accretion disks and galaxies. The reverse situation, with *Re* much larger than *Rm*, corresponds to planetary cores and the Sun's convective zone.

The new plasma experiment (figure 3e) under construction at Wisconsin will investigate dynamos and other magnetohydrodynamic phenomena in weakly magnetized, fast-flowing, hot plasmas. Plasma confinement in that device will be provided by a surface magnetic field generated by a high-order-multipole set of permanent magnets on the spherical vessel's surface. Recent simulations and prototype experiments have demonstrated the feasibility of driving von Kármán flow—trains of connected vortices—at the Wisconsin facility, and have shown that such flow might well generate a plasma dynamo.

There's much still to be done. Research opportunities in the investigation of natural magnetic dynamos encompass theory, computer simulation, experiment, and astronomical observation. That richly cross-disciplinary problem joins the forces with geophysics, astrophysics, fluid dynamics, nonlinear dynamics, and plasma physics.

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