Concepts from linear theory

Extra Lecture

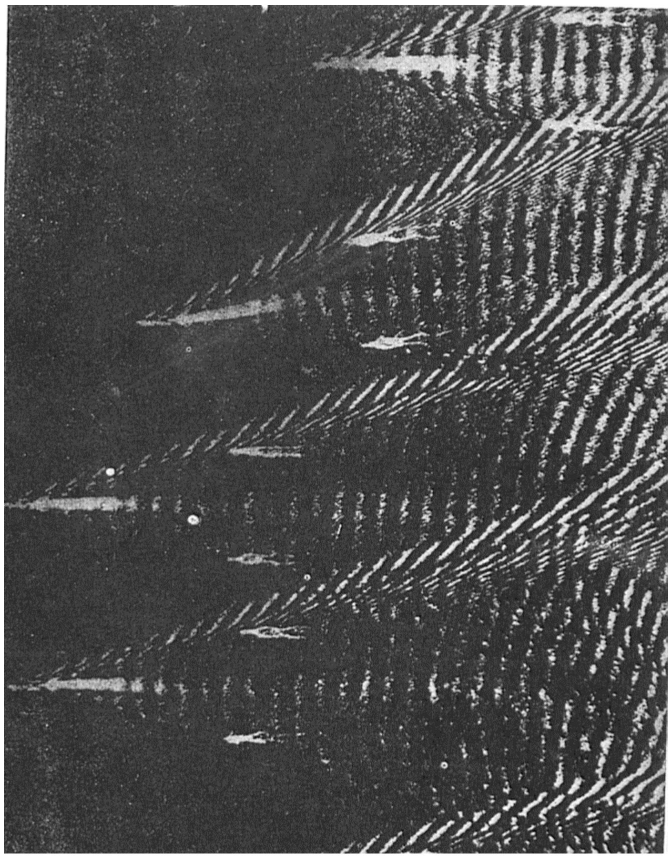

Fig. 5.9 – Ship waves on deep water. Photographing a model gives a more dramatic pattern than a full size ship. If the boat is not obviously a model the capillary waves in front show the scale clearly.

Ship waves from WW II battleships and a toy boat.

Kelvin’s (1887) method of stationary phase predicts both.

Concepts from linear theory

A. Linearize the nonlinear equations

B. Solve the linearized equations

C. Linearized dispersion relation
   - phase velocity
   - group velocity

D. Other predictions from linear theory
   - gravity waves and capillary waves
   - shallow water vs. deep water
   - paths of fluid particles

Nonlinear equations of motion

for an irrotational flow, with no forcing from wind:

$$\partial_t \eta + \nabla\phi \cdot \nabla\eta = \partial_z \phi, \qquad \text{on } z = \eta(x,y,t),$$

$$\partial_t \phi + \frac{1}{2}|\nabla\phi|^2 + g\eta = \frac{\sigma}{\rho}\nabla\cdot\{\frac{\nabla\eta}{\sqrt{1+|\nabla\eta|^2}}\}, \qquad \text{on } z = \eta(x,y,t),$$

$$\nabla^2\phi = 0 \qquad -h(x,y) < z < \eta(x,y,t),$$

$$\partial_z \phi + \nabla\phi \cdot \nabla h = 0, \qquad \text{on } z = -h(x,y).$$

Linearize these equations

about a trivial solution $\{\phi = 0,\ \eta = 0\}$

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Linearize these equations

about a trivial solution $\{\phi = 0,\ \eta = 0\}$

$$\partial_t \eta + \cancel{\nabla\phi \cdot \nabla\eta} = \partial_z \phi,$$ on $z = \eta(x,y,t)$,

$$\partial_t \phi + \cancel{\frac{1}{2}|\nabla\phi|^2} + g\eta = \frac{\sigma}{\rho}\nabla \cdot \{\frac{\nabla\eta}{\sqrt{1+\cancel{|\nabla\eta|^2}}}\},$$ on $z = \eta(x,y,t)$,

$$\nabla^2 \phi = 0$$ $-h(x,y) < z < \eta(x,y,t)$,

$$\partial_z \phi + \nabla\phi \cdot \nabla h = 0,$$ on $z = -h(x,y)$.

Linearize these equations

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$$\partial_t \eta + \cancel{\nabla\phi \cdot \nabla\eta} = \partial_z \phi, \qquad \text{on } z = \underset{0}{\cancel{\eta(x,y,t)}},$$

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$$\nabla^2 \phi = 0 \qquad -h(x,y) < z < \underset{0}{\cancel{\eta(x,y,t)}},$$

$$\partial_z \phi + \nabla\phi \cdot \nabla h = 0, \qquad \text{on } z = -h(x,y).$$

Linearized equations

For small amplitude waves on a flat bottom:

$$\partial_t \eta = \partial_z \phi, \qquad \partial_t \phi + g\eta = \frac{\sigma}{\rho}\nabla^2 \eta, \qquad \text{on } z = 0,$$

gravity, surface tension

$$\nabla^2 \phi = 0 \qquad -h < z < 0,$$

$$\partial_z \phi = 0, \qquad \text{on } z = -h.$$

Also need boundary conditions in (x,y), plus initial conditions.

Solve the linearized equations

1. Preliminary problems

   a) Find bounded $\phi(x,y,z,t)$ such that:

$$\phi = a \sin(kx) \qquad \text{on } z = 0,$$

$$\nabla^2\phi = 0 \qquad -h < z < 0,$$

$$\partial_z\phi = 0, \qquad \text{on } z = -h.$$

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Solution: $$\boxed{\phi = a \frac{\cosh(k(z+h))}{\cosh(kh)} \sin(kx)}$$

Solve the linearized equations

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   b) Change bottom boundary condition to

$$\phi \to 0 \quad \text{as} \quad z \to -\infty$$

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$\phi = a \sin(kx)$ on $z = 0$,

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$\partial_z\phi = 0,$ on $z = -h$.

Solution: $$\boxed{\phi = a\frac{\cosh(k(z+h))}{\cosh(kh)}\sin(kx)}$$

b) Change bottom boundary condition to

$\phi \to 0$ as $z \to -\infty$

Solution: $$\boxed{\phi = a \cdot e^{|k|z}\sin(kx)}$$

Solve the linearized equations

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$\phi = a \sin(kx)$ on $z = 0$,

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Solution: $$\boxed{\phi = a\frac{\cosh(k(z+h))}{\cosh(kh)}\sin(kx)}$$

c) Change top boundary condition to

$\phi = a \sin(kx) \cos(ly)$ on $z = 0$.

Solution:

Solve the linearized equations

1. Preliminary problems

   a) Find bounded $\phi(x,y,z,t)$ such that:

$$\phi = a \sin(kx) \qquad \text{on } z = 0,$$

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Solution: $\boxed{\phi = a\frac{\cosh(k(z+h))}{\cosh(kh)}\sin(kx)}$

   c) Change top boundary condition to

$$\phi = a \sin(kx)\cos(ly) \qquad \text{on } z = 0.$$

Solution: $\boxed{\phi = a\frac{\cosh(\kappa(z+h))}{\cosh(\kappa h)}\sin(kx)\cos(ly)}$, $\kappa^2 = k^2 + l^2$

Solve the linearized equations

2. Rewrite in complex notation

$$\phi = \mathrm{Re}\,\{a\ e^{ikx+ily}\} \qquad \text{on } z = 0,$$

$$\nabla^2\phi = 0 \qquad -h < z < 0,$$

$$\partial_z\phi = 0, \qquad \text{on } z = -h.$$

$$\boxed{\phi = \mathrm{Re}\{a\cdot\frac{\cosh(\kappa(z+h))}{\cosh(\kappa h)}e^{ikx+ily}\}, \quad \kappa^2 = k^2 + l^2.}$$

3. Use these functions to satisfy boundary conditions at $z = 0$

(the linearized approximation for the free surface).

Solve the linearized equations

3. Linearized problem

$$\partial_t \eta = \partial_z \phi, \qquad \partial_t \phi + g\eta = \frac{\sigma}{\rho}\nabla^2 \eta, \qquad \text{on } z = 0,$$

$$\nabla^2 \phi = 0 \qquad -h < z < 0,$$

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Solve the linearized equations

4. Linearized problem

$$\partial_t \eta = \partial_z \phi, \qquad \partial_t \phi + g\eta = \frac{\sigma}{\rho}\nabla^2 \eta, \qquad \text{on } z = 0,$$

$$\nabla^2 \phi = 0 \qquad -h < z < 0,$$

$$\partial_z \phi = 0, \qquad \text{on } z = -h.$$

Substitute in one Fourier mode:

$$\phi(x,y,z,t;...) = \Phi(k,l,\omega)\frac{\cosh(\kappa(z+h))}{\cosh(\kappa h)} e^{ikx+ily-i\omega t},$$

$$\eta(x,y,z,t;...) = H(k,l,\omega) e^{ikx+ily-i\omega t}.$$

Solve the linearized equations

4. Linearized equations at free surface:

$$\partial_t \eta = \partial_z \phi, \qquad \partial_t \phi + g\eta = \frac{\sigma}{\rho}\nabla^2 \eta, \quad \text{on } z = 0,$$

Find linearized dispersion relation:

$$\boxed{\omega^2 = (g + \frac{\sigma}{\rho}\kappa^2)[\kappa \cdot \tanh(\kappa h)], \quad \kappa^2 = k^2 + l^2.}$$

Frequency gravity surface tension water depth wavenumbers

Solve the linearized equations

5. Linearized dispersion relation

$$\boxed{\omega^2 = (g + \frac{\sigma}{\rho}\kappa^2)[\kappa \cdot \tanh(\kappa h)], \quad \kappa^2 = k^2 + l^2.}$$

General fact: If a system of linear evolution equations has a dispersion relation, it encodes all the important information about wave propagation in those equations.

Solve linearized equations in 2-D

5. Linearized dispersion relation in 2-D $(\partial_y \equiv 0)$

$$\omega^2 = (gh + \frac{\sigma}{\rho} k^2 h)[\frac{\tanh(kh)}{kh}] \cdot k^2$$

Define

$$\omega(k) = \sqrt{gh + \frac{\sigma}{\rho} k^2 h} \cdot \sqrt{\frac{\tanh(kh)}{kh}} \cdot k$$

so

$$\frac{\omega(k)}{k} \geq 0$$

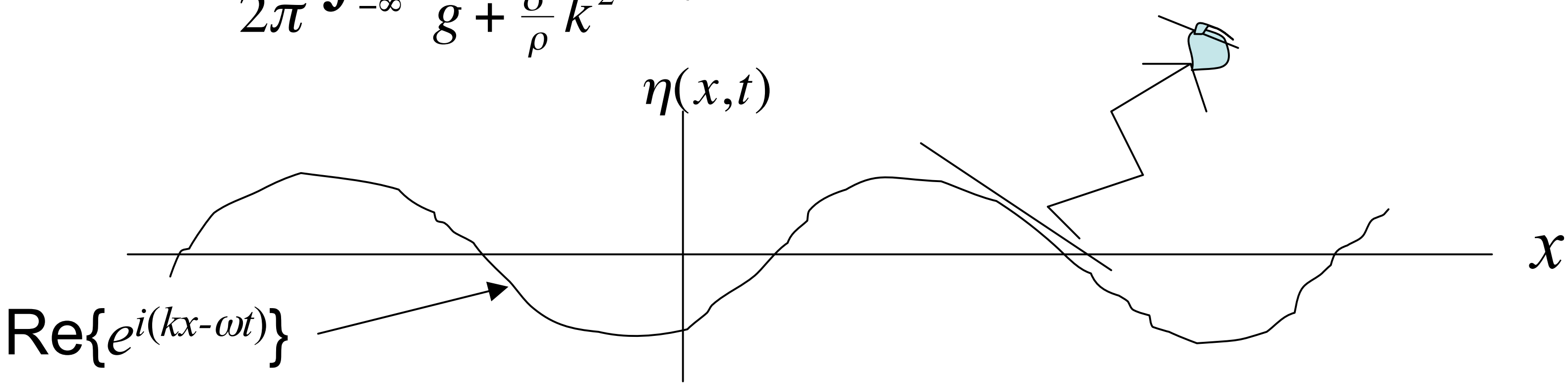
Then construct general solution of linearized problem in 2-D:

Linearized solution in 2-D, $\frac{\omega(k)}{k} \geq 0$

$$\phi(x,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{-}(k)\frac{\cosh(k(z+h))}{\cosh(kh)} e^{i(kx-\omega(k)t)}dk$$
$$+\frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{+}(k)\frac{\cosh(k(z+h))}{\cosh(kh)} e^{i(kx+\omega(k)t)}dk,$$

right-going waves

$$\eta(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\frac{i\omega(k)}{g+\frac{\sigma}{\rho}k^2}\Phi_{-}(k)]e^{i(kx-\omega(k)t)}dk$$
$$-\frac{1}{2\pi}\int_{-\infty}^{\infty} [\frac{i\omega(k)}{g+\frac{\sigma}{\rho}k^2}\Phi_{+}(k)]e^{i(kx+\omega(k)t)}dk.$$



B. Linearized dispersion relation, 2-D

1. The dispersion relation

$$\omega(k) = \sqrt{gh + \frac{\sigma}{\rho}k^2 h} \cdot \sqrt{\frac{\tanh(kh)}{kh}} \cdot k$$

For a right-going wave,

$$\eta(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\frac{i\omega(k)}{g + \frac{\sigma}{\rho}k^2}\Phi_{\_}(k)]e^{i(kx-\omega(k)t)}dk$$

Define the phase velocity, $\boxed{c_p(k) = \frac{\omega(k)}{k} \geq 0}$

For each k, its wave crests move with speed $c_p(k)$.

Linearized dispersion relation, 2-D

2. For any dispersion relation of one variable, $\omega(k)$,

$$\boxed{c_p(k) = \frac{\omega(k)}{k},} \qquad \boxed{c_g(k) = \frac{d\omega}{dk}}$$

define phase velocity group velocity.

Linearized dispersion relation, 2-D

2. For any dispersion relation of one variable, $\omega(k)$,

$$\boxed{c_p(k) = \frac{\omega(k)}{k}}, \qquad \boxed{c_g(k) = \frac{d\omega}{dk}}$$

define phase velocity group velocity.

a) For wave equation in 1-D, $\omega = ck$ ➔ $c_p = c_g = c$

All waves travel with same speed

➔ nondispersive.

Linearized dispersion relation, 2-D

2. For any dispersion relation of one variable, $\omega(k)$,

$$\boxed{c_p(k) = \frac{\omega(k)}{k},} \qquad \boxed{c_g(k) = \frac{d\omega}{dk}}$$

define phase velocity group velocity.

a) For wave equation in 1-D, $\omega = ck$ ➔ $c_p = c_g = c$

All waves travel with same speed

➔ nondispersive.

b) If $\frac{d^2\omega}{dk^2} \neq 0,$ then waves with different k move with different speeds ➔ dispersive.

c) Water waves are dispersive.

Significance of group velocity

Add two wave modes, with slightly different wave numbers: $\{k,\, k + \delta k\}$ ($\delta k << k$)

c_g

and frequencies: $\{\ \omega(k),\ \ \omega(k+\delta k) \sim \omega(k) + \frac{d\omega}{dk} \cdot \delta k\ \}$

$$\eta(x,t) = \sin\{kx - \omega t\} + \sin\{(k+\delta k)x - (\omega + c_g \delta k)t\}$$

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$$\eta(x,t) = \sin\{kx - \omega t\} + \sin\{(k+\delta k)x - (\omega + c_g \delta k)t\}$$

$$= 2\sin\{(k + \frac{\delta k}{2})x - (\omega + \frac{c_g \delta k}{2})t\} \cdot \cos\{\frac{\delta k}{2}x - \frac{c_g \delta k}{2}t\}$$

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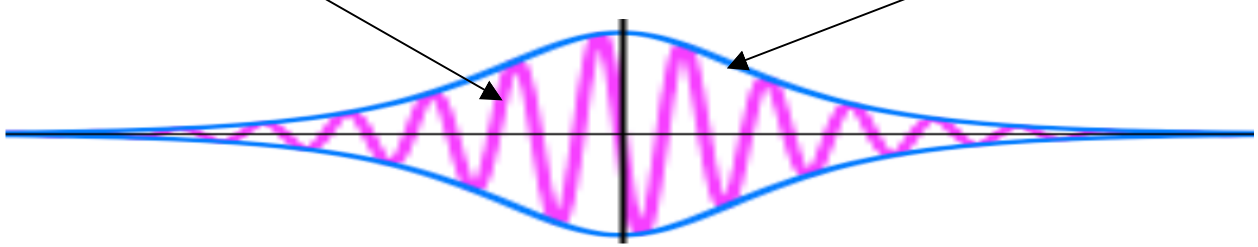
$$\Rightarrow \quad \boxed{\eta(x,t) \approx 2\sin\{k(x - c_p t)\} \cdot \cos\{\frac{\delta k}{2}(x - c_g t)\}}$$

fast oscillation

slow modulation



Significance of group velocity

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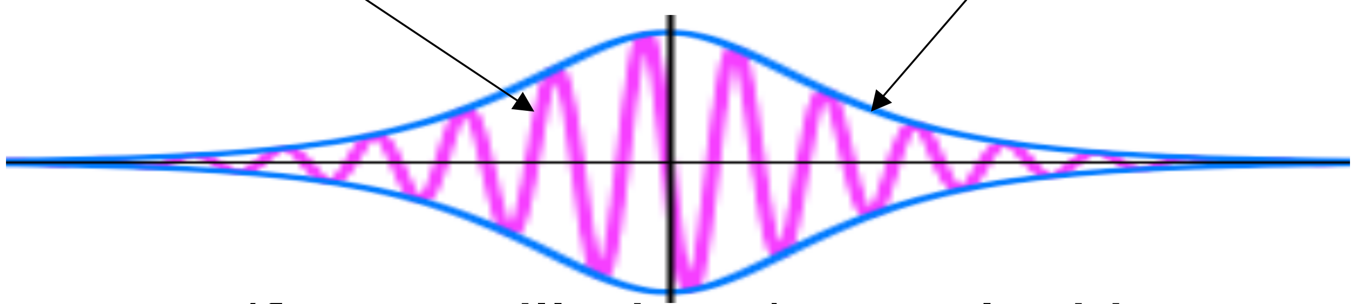

Wave crests (fast oscillations) travel with
the phase velocity

The wave packet (slow modulation) travels with
the group velocity

(see wikipedia movie: “group velocity”)

Significance of group velocity

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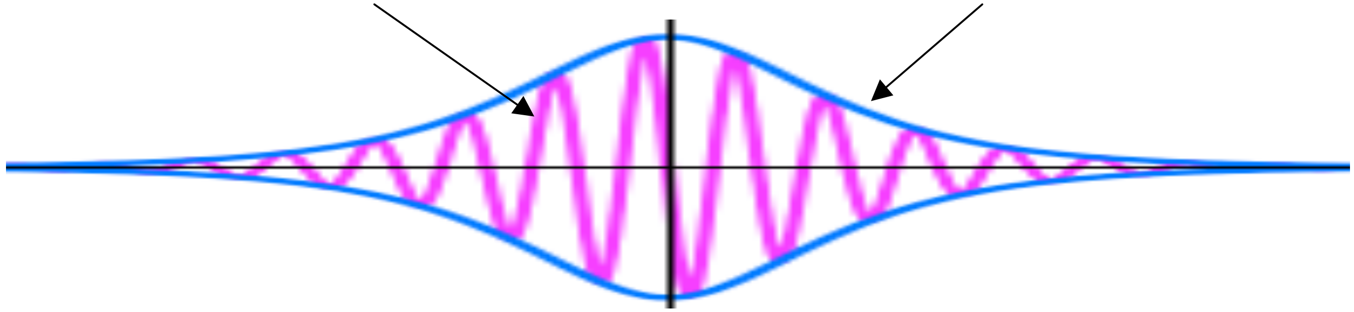
fast oscillation

slow modulation



If $c_p \neq c_g$, the largest wave in a wave group only dominates for a limited time

➔ surfing is impossible for very dispersive waves.

Surfing only works where $c_p \approx c_g$

Significance of group velocity

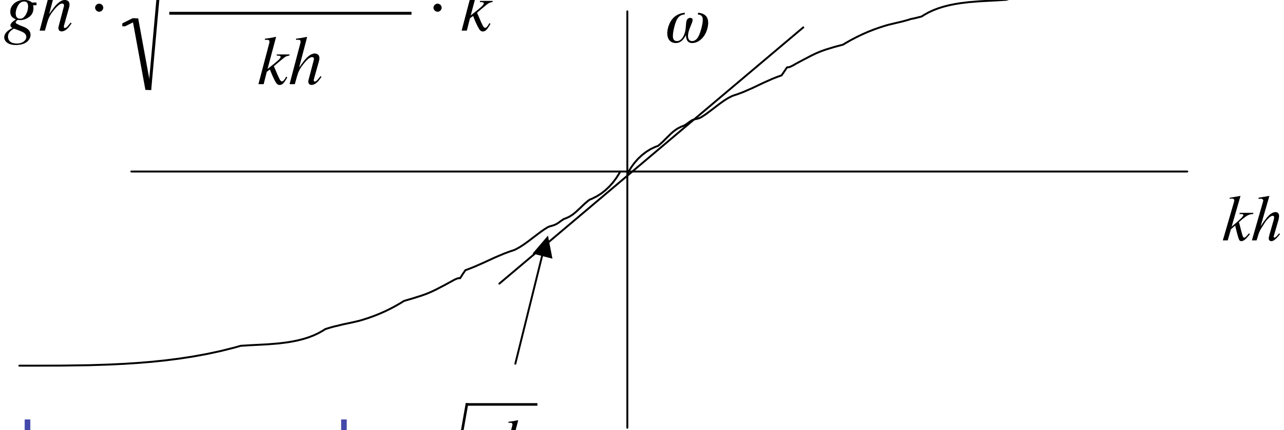
Another argument that also shows that wave packets travel with the group velocity is based on Kelvin's (1887) method of stationary phase. That line of reasoning leads to concrete formulae for the long-time behaviour of a dispersive wave system.

[See homework set 2.5.]

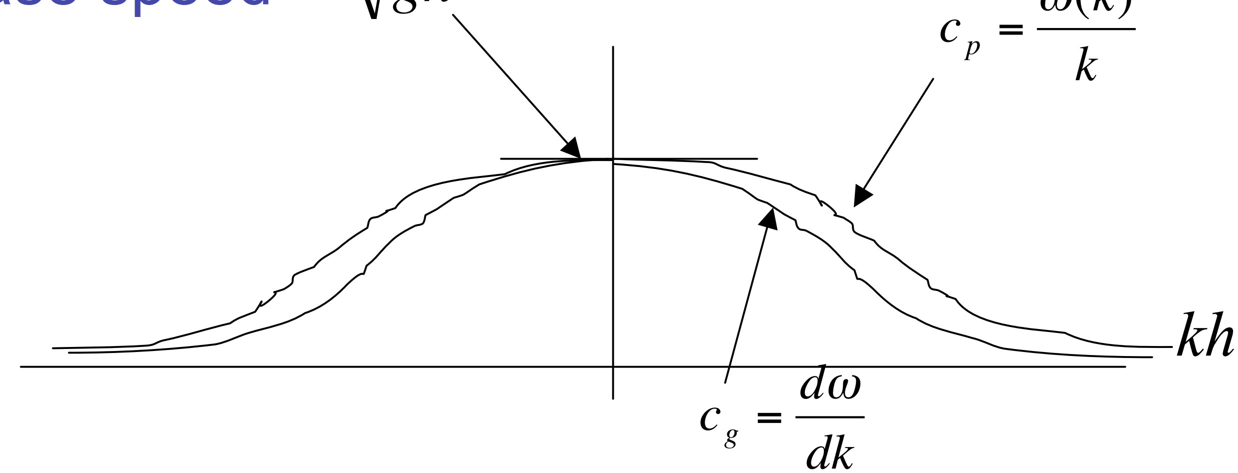
Linearized dispersion relation, 2-D

3. With no surface tension (“gravity waves”)

$$\omega(k) = \sqrt{gh} \cdot \sqrt{\frac{\tanh(kh)}{kh}} \cdot k$$



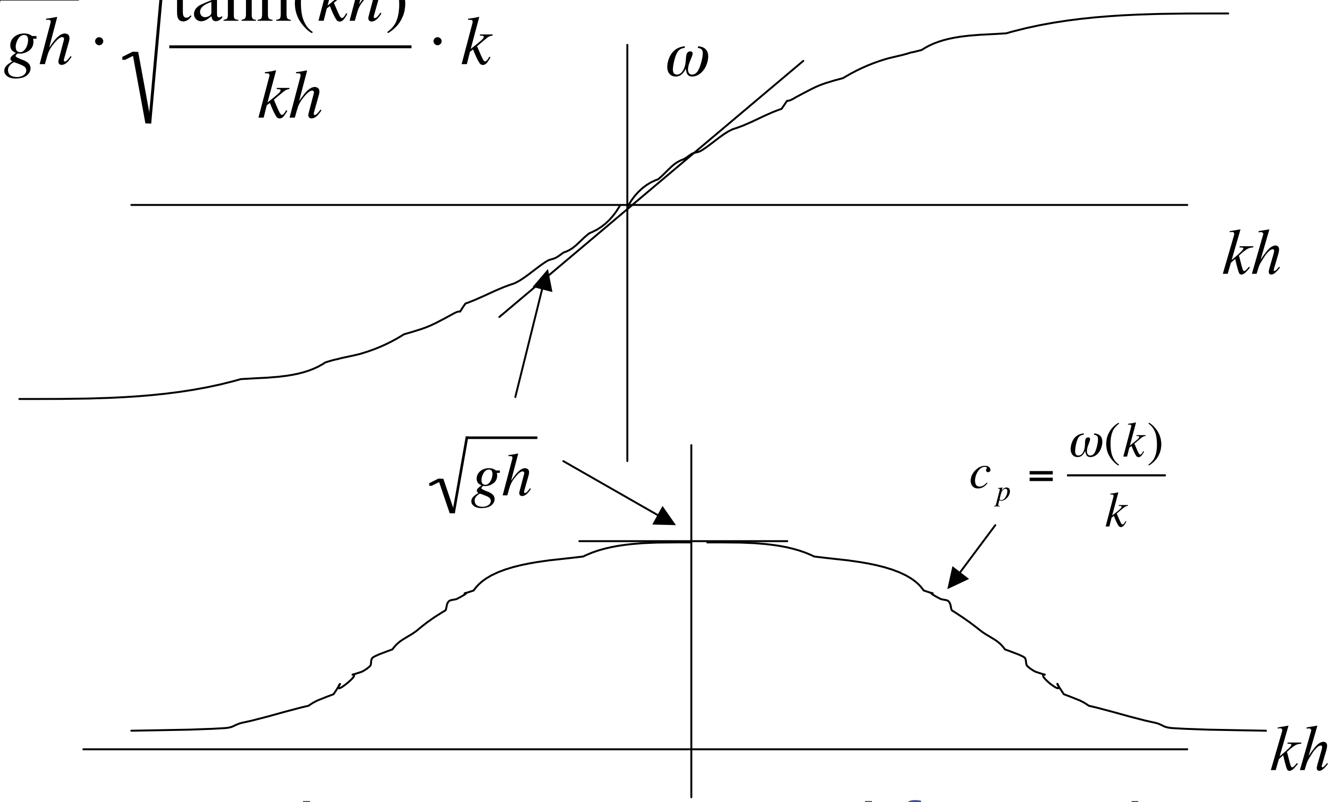
Max phase speed = $\sqrt{gh}$



C. Predictions from linear theory, 2-D

1. If no surface tension ("gravity waves")

$$\omega(k) = \sqrt{gh} \cdot \sqrt{\frac{\tanh(kh)}{kh}} \cdot k$$



For gravity waves, long waves travel faster than short waves (recall Stoker's "video")

How fast can ocean waves travel?

2. Max phase speed of gravity waves = $\sqrt{gh}$

- Height of Mt. Everest – 8848 m
- Deepest point in ocean (near Guam) – 11,000 m
- Fastest gravity wave in ocean – 328 m/sec

= 1182 km/hr = 734 mi/hr

- Speed of sound in air (at 10°C) = 340 m/sec
- Speed of sound in water (at 10°C) = 1450 m/sec

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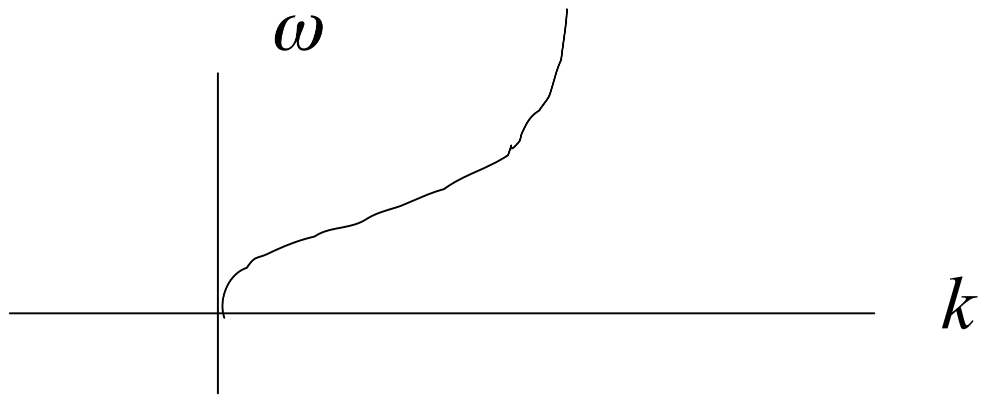
- Speed of sound in water (at 10°C) = 1450 m/sec
- Depth of Bay of Bengal (tsunami) – 3500 m
- Speed of tsunami – 185 m/sec = 670 km/hr = 415 mph

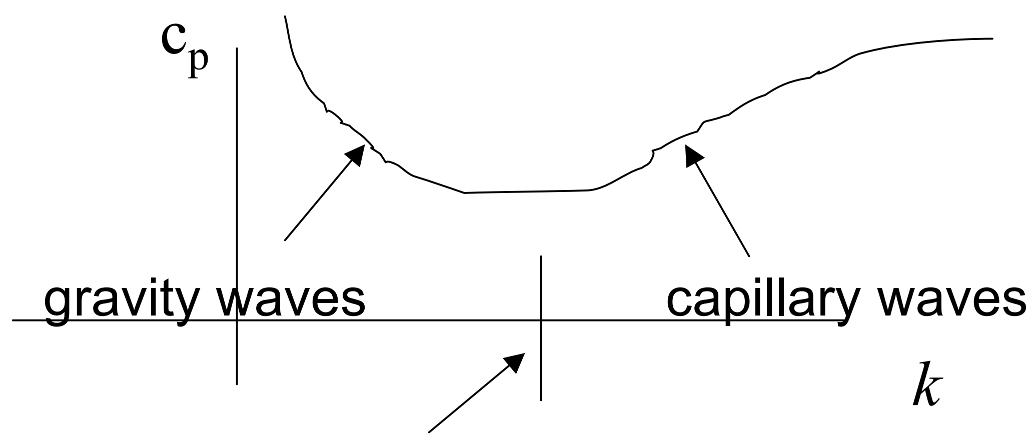
Predictions from linear theory, 2-D

3. With surface tension, in deep water

$$\omega^2 = \left(g + \frac{\sigma}{\rho} k^2\right) |k|$$



$$c_p = \frac{\omega}{k} = \frac{\sqrt{g + \frac{\sigma}{\rho} k^2}}{\sqrt{|k|}}$$



minimum phase speed at L = 1.73 cm

Predictions from linear theory, 2-D

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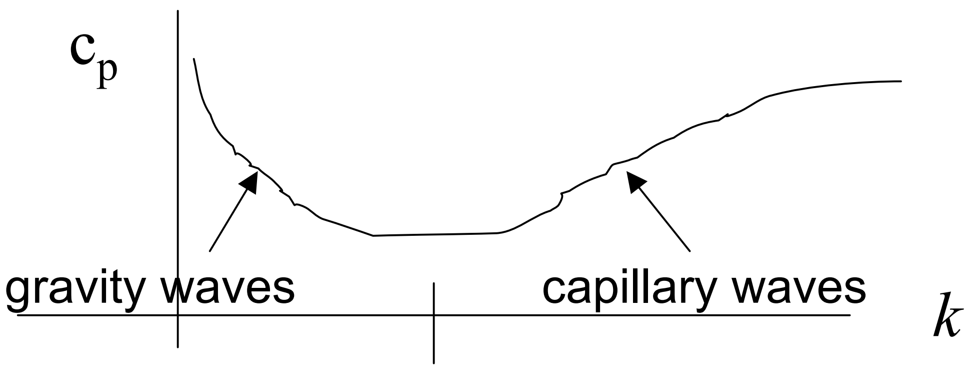
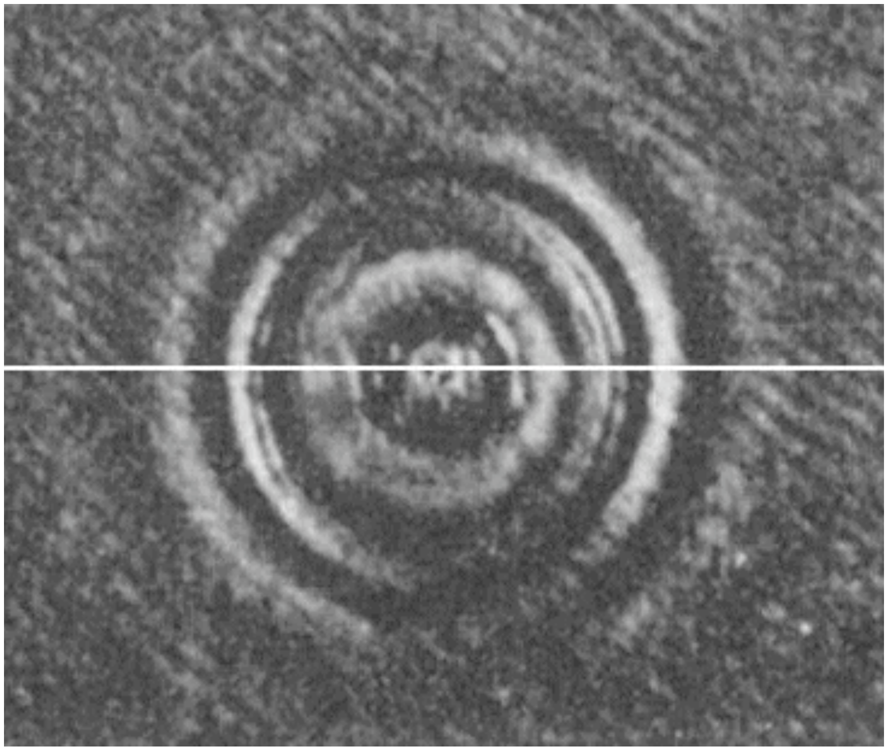
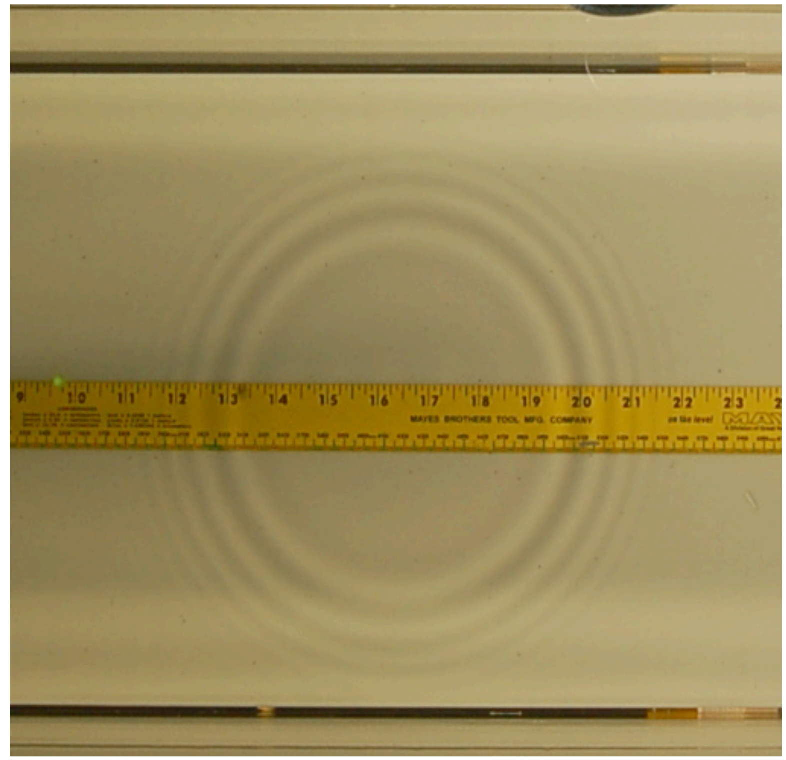


- For gravity waves, long waves travel faster than short waves. (recall Stoker's "video")
- For capillary waves, the opposite happens.
- Including surface tension guarantees a minimum phase speed, and a minimum group speed

Predictions from linear theory, 2-D

3. With surface tension, in deep water





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Predictions from linear theory, 2-D

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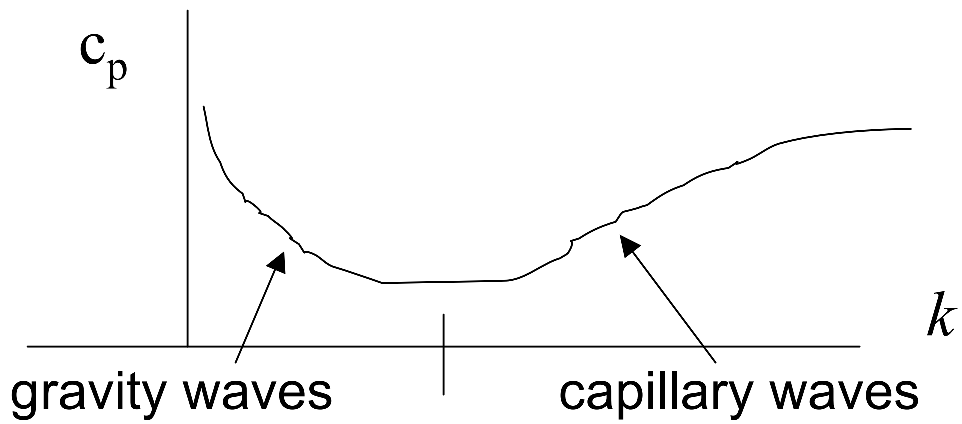
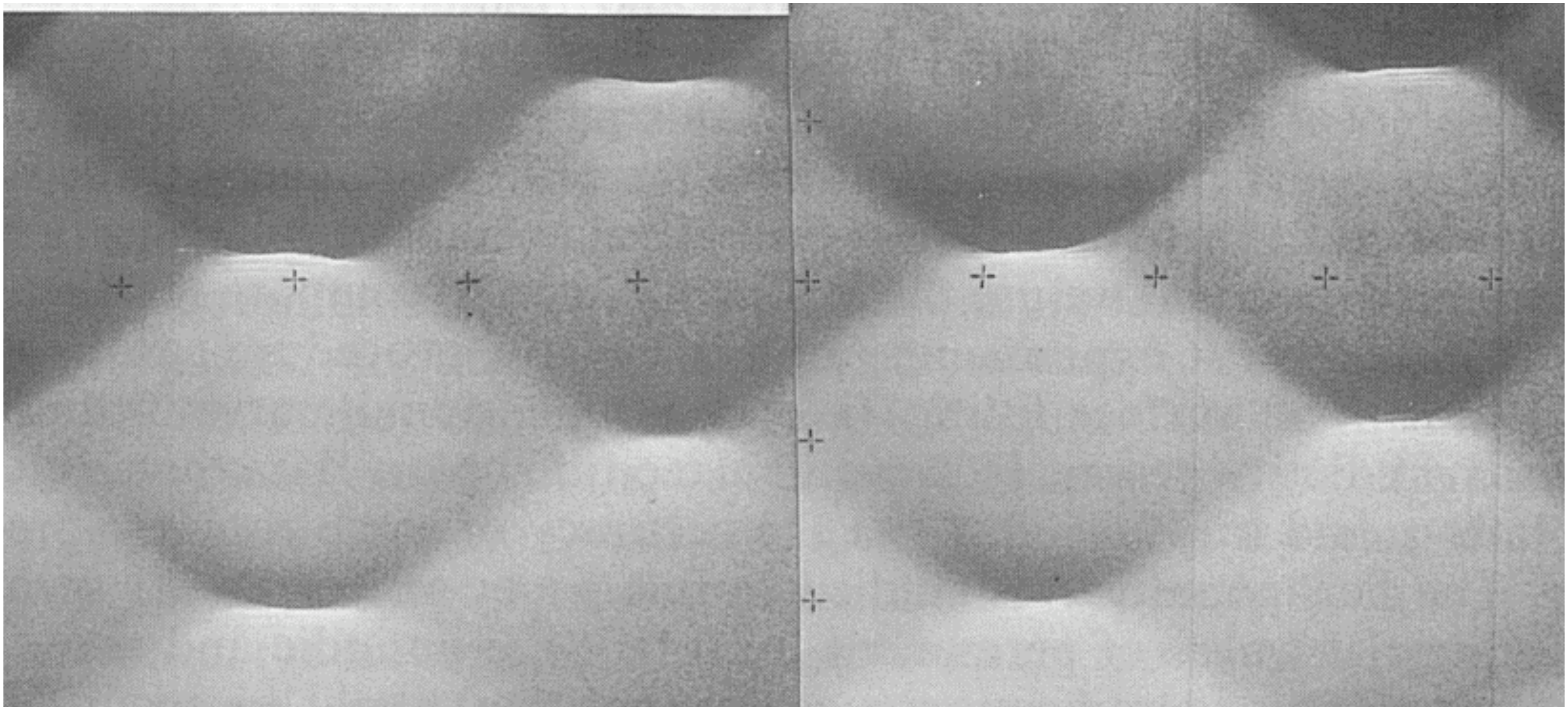


- For every (long) gravity wave, there is a (short) capillary wave with the same phase speed
  [Remote sensing of ocean waves depends on this.]



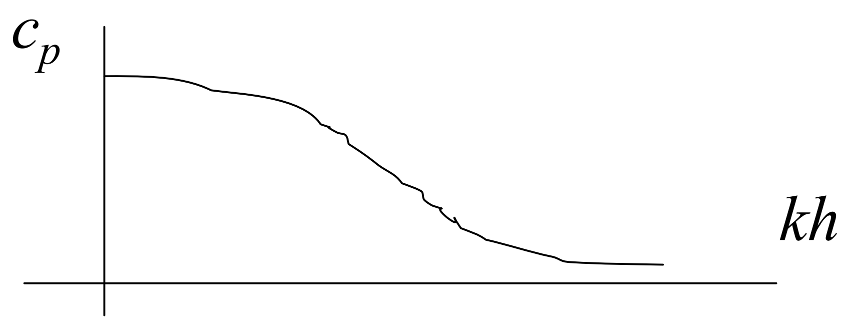
A 2-dimensional surface pattern of nearly permanent form, propagating in shallow water. Small capillary waves can be seen on the front faces of the steepest gravity waves.

(Hammack *et al*, 1995)

Predictions from linear theory, 2-D

4. Back to gravity waves (no surface tension)

$$c_p = \frac{\omega}{k} = \sqrt{gh}\sqrt{\frac{\tanh(kh)}{kh}}$$



"shallow water" $\leftrightarrow$ "long waves": $kh << 1, \quad c_p \to \sqrt{gh}$

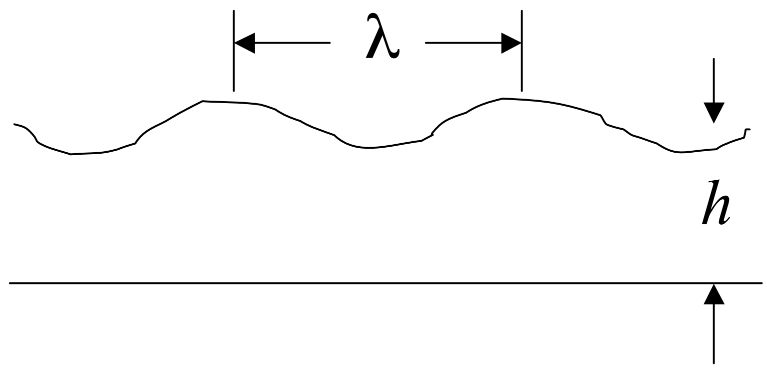
"deep water" $\longleftrightarrow$ "short waves: $kh >> 1, \quad c_p \to \sqrt{\frac{g}{|k|}}$

What does "deep" mean?

Predictions from linear theory, 2-D

5. Gravity waves in "deep water"

$$c_p = \frac{\omega}{k} = \sqrt{gh}\sqrt{\frac{\tanh(kh)}{kh}}$$

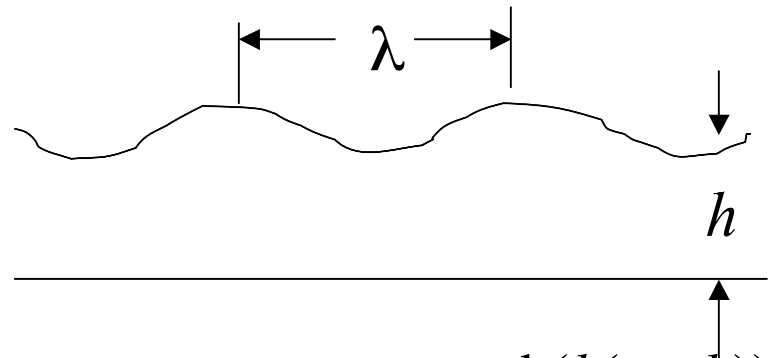


Suppose $\lambda = h \Rightarrow kh = 2\pi h/\lambda = 2\pi$

$\Rightarrow \tanh(kh) = 0.999993$

Predictions from linear theory, 2-D

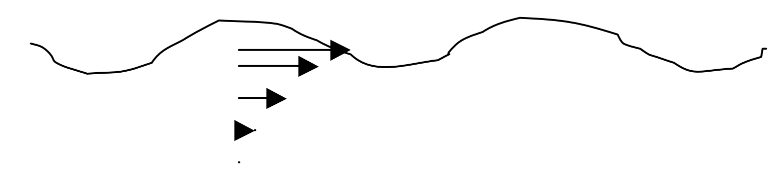
5. Gravity waves in "deep water"



$\lambda = h \Rightarrow \; kh = 2\pi$

Horizontal velocity: $$u(x,z,t) = a\sin(kx - \omega(k)t)\frac{\cosh(k(z+h))}{\cosh(kh)}$$

$$\frac{u(z=-h)}{u(z=0)} = 0.0037$$



➔ Only long waves create motion at bottom

(relevant for fishing, and garbage disposal)

Particle paths, from linear theory

6. For a right-going wave, wave crests move to right with fixed speed, $c_p(k)$.

   Where do fluid particles go?

$$\frac{Dx}{Dt} = u(x,z,t) = a\cos(kx - \omega(k)t)\frac{\cosh(k(z+h))}{\cosh(kh)},$$

$$\frac{Dz}{Dt} = w(x,z,t) = a\sin(kx - \omega(k)t)\frac{\sinh(k(z+h))}{\cosh(kh)}.$$

Nonlinear ODEs (hard to solve)

Approximate: Assume $x(t)$ and $z(t)$ do not change much.

Particle paths, from linear theory

$$\frac{Dx}{Dt} = u(x,z,t) \sim a\cos(kx_0 - \omega(k)t)\frac{\cosh(k(z_0+h))}{\cosh(kh)},$$
$$\frac{Dz}{Dt} = w(x,z,t) \sim a\sin(kx_0 - \omega(k)t)\frac{\sinh(k(z_0+h))}{\cosh(kh)}.$$

$$x(t) \sim x_0 - \frac{a}{\omega}\sin(kx_0 - \omega(k)t)\frac{\cosh(k(z_0+h))}{\cosh(kh)},$$
$$\Rightarrow \quad z(t) \sim z_0 + \frac{a}{\omega}\cos(kx_0 - \omega(k)t)\frac{\sinh(k(z_0+h))}{\cosh(kh)}.$$

Particle paths, from linear theory

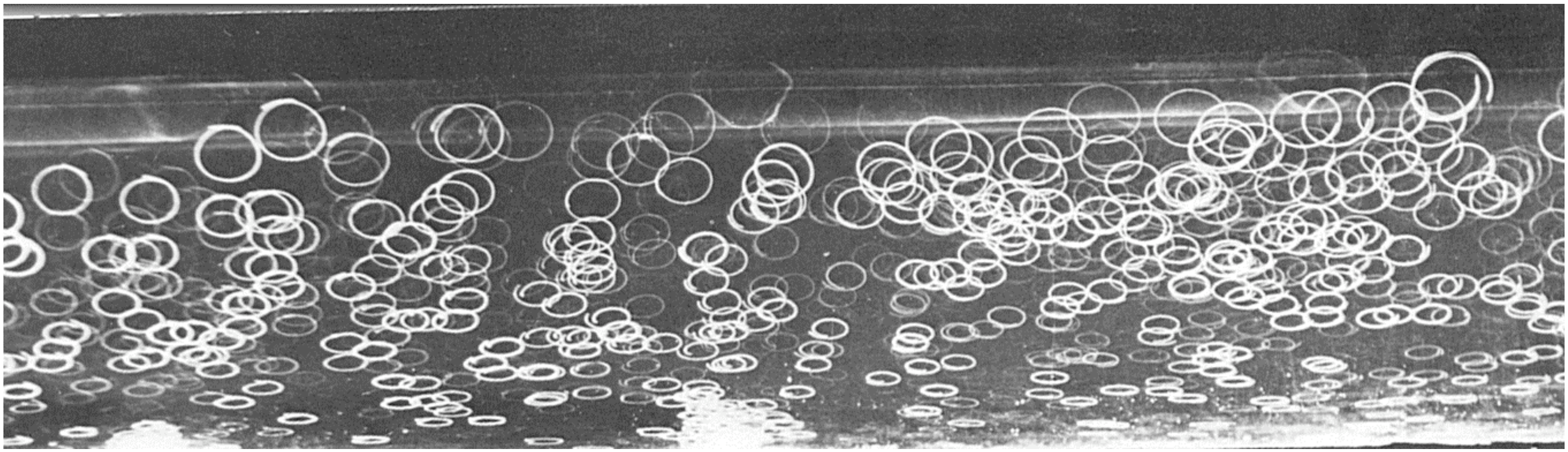
$$\frac{Dx}{Dt} = u(x,z,t) \sim a\cos(kx_0 - \omega(k)t)\frac{\cosh(k(z_0+h))}{\cosh(kh)},$$
$$\frac{Dz}{Dt} = w(x,z,t) \sim a\sin(kx_0 - \omega(k)t)\frac{\sinh(k(z_0+h))}{\cosh(kh)}.$$

➔
$$x(t) \sim x_0 - \frac{a}{\omega}\sin(kx_0 - \omega(k)t)\frac{\cosh(k(z_0+h))}{\cosh(kh)},$$
$$z(t) \sim z_0 + \frac{a}{\omega}\cos(kx_0 - \omega(k)t)\frac{\sinh(k(z_0+h))}{\cosh(kh)}.$$

➔
$$\frac{(x(t)-x_0)^2}{\cosh^2(k(z_0+h))} + \frac{(z(t)-z_0)^2}{\sinh^2(k(z_0+h))} \sim const$$

➔ elliptical orbit (to first approximation)

Particle paths, from linear theory



Motion of marked fluid particles over one wave period. Orbits are nearly circular near the top of this flow, and approximately elliptical at every level. From Wallet & Ruellen, 1950.

Stokes showed that at second order, there is a slow drift near the free surface (“Stokes drift”). The drift is visible in this photo.

End of summary of linear theory

Lecture 3 explores the Hamiltonian nature of the (nonlinear) water wave equations.