A Study of Heat Transport and the Runaway Greenhouse Effect using an Idealized Model

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1 Introduction

The Earth, in its current state, is in a delicate balance of incoming radiation from the sun, and outgoing radiation emitted by our planet. For the Earth to maintain a certain climate, the amount of incoming and outgoing radiation must be equal, and thus we say that the Earth is in radiative equilibrium. What would happen to the temperature on Earth if something were to perturb our current equilibrium? This work looks into the possibility of a runaway greenhouse effect and how an increase in the incoming solar radiation would affect the temperature of the Earth's surface. We use a one- and two-column model, first in pure radiative equilibrium, then with added convection, and allow the atmosphere's optical depth to vary with temperature and allow for lateral heat transport between the two columns. Our goal is to investigate whether the lateral heat transport from the equator toward the poles might mitigate the impact of the runaway greenhouse effect.

2 Brief Background

The majority of the Earth's energy comes from the incoming solar radiation, which passes through most of the components of the atmosphere, straight to the ground. This light is reflected back upwards through the atmosphere, largely in the form of infrared radiation, some of which gets absorbed by the atmosphere. The absorbed radiation thus heats up the atmosphere, and this is why the Earth's surface temperature is warm enough for human habitation [9] [3]. This atmospheric warming is called a greenhouse effect.

In this work, we will investigate the runaway greenhouse effect. This phenomenon occurs when the Earth absorbs more radiation than it can emit, and thus is no longer in equilibrium. If the surface temperature of the planet increases, more water vapor is formed in the atmosphere, which then traps more of the upward infrared radiation, leading to an even higher concentration of water vapor in the atmosphere, and so the cycle continues, and hence the term "runaway" [4] [5] [9]. In Ingersoll (1969), it is proposed that Venus underwent such a runaway greenhouse effect that eventually caused its oceans to boil away [4]. Here, we take a preliminary look into how close the Earth is to experiencing the same fate as Venus.

Symbol	Meaning	Value
S_0	solar constant	$1366\ W/m^2$
α	albedo	0.3
σ	Stefan-Boltzmann constant	$5.67 * 10^{-8}$
T_g	ground temperature	
T_a	atmospheric temperature	

Table 1: Table of symbols used in this section.

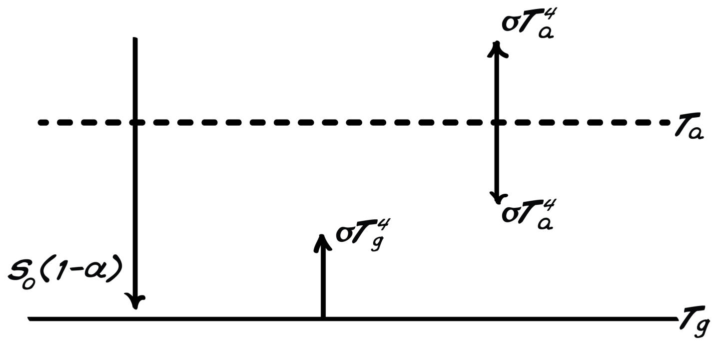

Figure 1: A one-column, two-layer model of radiative balance.

3 The Model

3.1 One-Column Radiative Equilibrium

(Note: the following derivations are obtained by following the steps in Geoffrey Vallis' 2014 GFD Lecture Notes [9]). We first consider a very simple radiative equilibrium model of the atmosphere, starting with a single column, two-layer setup. As illustrated in figure 1, the only radiative fluxes present are the incoming solar radiation, $S_0(1-\alpha)$, the upward heat flux from the ground, given by the Stefan-Boltzmann Law, σT_g^4, and the upward and downward heat fluxes emitted from the atmosphere layer, σT_a^4 [7] [1] [8] [6]. (See table 1 for the meanings and values of the symbols used in this and following sections.) To keep the notation consistent, it is common to represent the incoming solar radiation in the form of the Stefan-Boltzmann Law in terms of an effective emitting temperature of the Earth, which we call T_e. In equation form, this gives [6] [2]

$$\sigma T_e^4 = S_0(1-\alpha). \tag{1}$$

The equations for the model illustrated in figure 1 can be written assuming radiative equilibrium at each layer - the ground and the atmosphere.

At the surface of the Earth:

$$\sigma T_e^4 + \sigma T_a^4 = \sigma T_g^4 \tag{2a}$$

At the top of the atmosphere:

$$\sigma T_e^4 = \sigma T_a^4 \tag{2b}$$

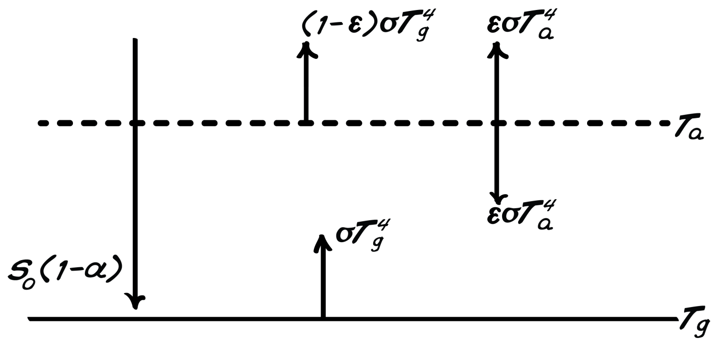

Figure 2: A one-column, two-layer model of radiative balance with an absorbing atmosphere.

This clearly gives $T_a = T_e$, and thus $T_g = 2^{\frac{1}{4}} T_e$. For an emitting temperature $T_e = 255K$, this implies a ground temperature of about $T_g = 303K$, which overestimates the actual ground temperature of around $288K$, so we choose to improve on this model.

The model assumes that the Earth is a perfect black body and thus absorbs no radiation. A small tweak to this model yields slightly more realistic results; we assume that the atmosphere boundary emits only a fraction ϵ (called the "emissivity") of the incoming radiation flux (see figure 2) [2]. Again, balancing incoming and outgoing radiation at each of the two layers in the model gives the following equations.

At the surface

$$\sigma T_e^4 + \epsilon \sigma T_a^4 = \sigma T_g^4 \tag{3a}$$

At the top

$$\sigma T_e^4 = \epsilon \sigma T_a^4 + (1-\epsilon)\sigma T_g^4. \tag{3b}$$

We now have a set of equations that depend on the parameter ϵ, which varies between 0 and 1. However, this restriction on the value of ϵ is too strict to allow for a runaway greenhouse effect , and so we chose to upgrade this model to one that is continuous in the vertical direction. The new setup is still in one-column form, but instead of having layers and setting up radiative balance at the interfaces for specific values of the emissivity, we have a continuous atmosphere written in terms of the optical depth τ. The emissivity and optical depth are essentially measuring the same quantity - how much of the incoming radiation gets transmitted, i.e. a measure of the atmosphere's opacity to long wave radiation. The equations in this case are more complicated, and cannot be simply read directly from the diagram, but are derived from the radiative transfer equations.

3.1.1 Radiative Transfer Equations

Let's begin by considering a beam of radiating particles traveling through a medium that is also emitting radiation. The intensity I of the beam of particles changes according to the equation

$$dI = (B - I)d\tau, \tag{4}$$

where B is the radiation emitted by the medium, given by the Stefan-Boltzmann Law (σT^4), and τ is the optical depth. Applying this equation to the atmosphere, and assuming a two-stream atmosphere, we allow for only upward (U) and downward (D) radiation. We define τ in the standard way, i.e. decreasing with atmospheric height such that $\tau = 0$ at the top of the atmosphere. The equations are thus

$$dU = -(B - U)d\tau \tag{5a}$$

$$dB = (B - D)d\tau. \tag{5b}$$

Notice the negative sign in the first equation, since the upward radiation U increases with decreasing τ. In the atmosphere, B refers to σT^4 for the T at a certain height, dictated by the value of τ. Dividing both sides by $d\tau$, we are left with the differential equations

$$\frac{dU}{d\tau} = U - B \tag{6a}$$

$$\frac{dB}{d\tau} = B - D. \tag{6b}$$

It turns out to be convenient to change variables to $(U + D)$ and $(U - D)$, which gives

$$\frac{d(U + D)}{d\tau} = U - D \tag{7a}$$

$$\frac{d(U - D)}{d\tau} = U + D - 2B. \tag{7b}$$

$$\frac{d(U - D)}{d\tau} = 0 \tag{8}$$

These are the equations that we would like to find solutions for, imposing the boundary conditions

$$D = 0 \tag{9a}$$

$$U = \sigma T_e^4 \tag{9b}$$

for $\tau = 0$.

The solution for the given boundary condition is the following:

$$D = \sigma T_e^4 \left(\frac{\tau}{2}\right) \tag{10a}$$

$$U = \sigma T_e^4 \left(1 + \frac{\tau}{2}\right) \tag{10b}$$

$$B = \sigma T_e \left(\frac{1 + \tau}{2}\right). \tag{10c}$$

The final equation is the one we are interested in - the one that relates the temperature of the atmosphere (in B) to the emitting temperature (T_e).

We are interested in the ground temperature of the Earth, but so far we have not accounted for a black surface at the ground level when z=0. From equation 10, we know

Symbol	Value
a	1.12
b	.14
c	5413

Table 2: The values of the constants used in the expansion of optical depth in terms of temperature.

the expression for the upward radiation. This is true at all levels, and so at z=0 with temperature T_g, we have

$$T_g^4 = \left(\frac{2+\tau}{1+\tau}\right) T_s^4, \tag{11}$$

where I use T_s to denote the temperature of the surface, i.e. just above the actual ground. Below we plug this equation in to replace T_s with T_g, the ground temperature which is the desired value.

The next step is to expand the optical depth in terms of temperature. We write the optical depth first in terms of z:

$$\tau = \tau_0 e^{-z/H_a}, \tag{12}$$

where z is the vertical height (in km) and the constant H_a corresponds to the characteristic height of water vapor in the atmosphere, which is taken to be 2 km. Because this work is studying the temperature only at the surface of the Earth, we take $z = 0$. However, we expand τ_0 as a function of temperature, defined as

$$\tau_0(T_g) = a + b\left(e^{c\left(\frac{1}{288}-\frac{1}{T_g}\right)}\right). \tag{13}$$

The constants a,b, and c (see Table 2) are determined by model tuning in conjunction with discussions with Andy Ingersoll.

With $z = 0$, and plugging in for τ and T_g, the final equation is

$$\sigma T_g^4 = \sigma T_e^4 \left(1 + \frac{a + b\left(e^{c\left(\frac{1}{288}-\frac{1}{T_g}\right)}\right)}{2}\right). \tag{14}$$

The solutions to this equation for various emitting temperatures, as well as other analysis, are shown in the Results Section.

Before adding convection to the model, we first add a term to the equation that allows for additional heat loss, and that will lead into the two-column model that will be discussed later. The new term takes the form $-kT$ and yields the slightly altered equation:

$$\sigma T_g^4 = (\sigma T_e^4 - kT_g)\left(1 + \frac{a + b\left(e^{c\left(\frac{1}{288}-\frac{1}{T_g}\right)}\right)}{2}\right). \tag{15}$$

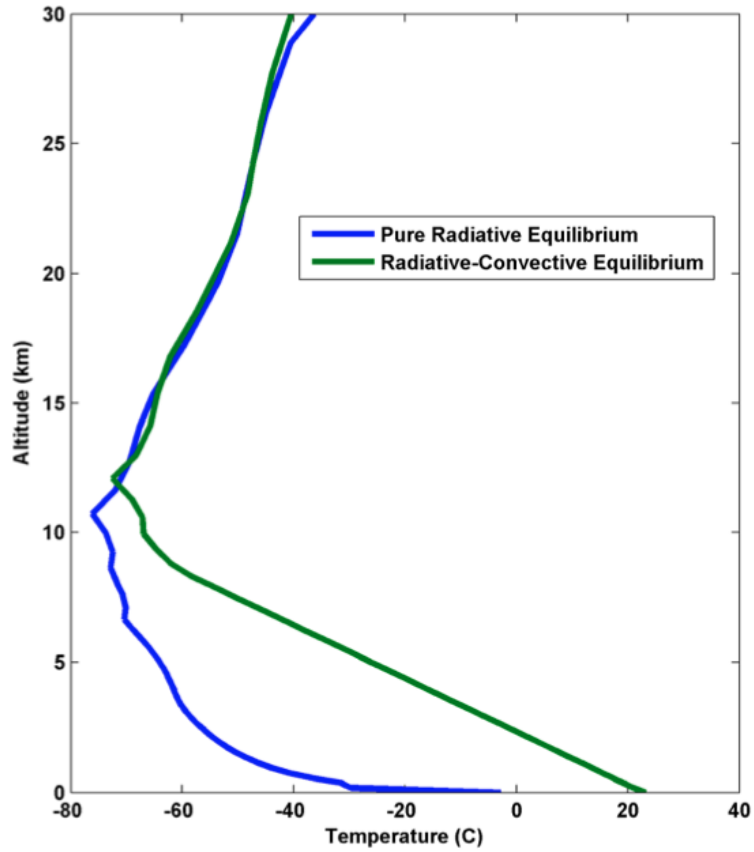

Figure 3: The temperature profiles of radiative and radiative-convective equilibrium. (Graph by Kerry Emanuel [2])

In the next section we will allow for atmospheric convection to occur, in addition to radiation.

3.2 One-Column Radiative-Convective Equilibrium

Convection plays an important role in the atmosphere, particularly in the troposphere, and greatly influences the heating in the lower atmosphere, as shown in figure 3. From this graph, the temperature profile in the lower atmosphere clearly changes with the addition of convection; instead of a large increase in temperature very close to the ground in the purely radiative case, the temperature change is roughly linear with both radiative and convective effects [2]. Convection, therefore, is an important process to include in our model.

The approach differs from the purely radiative case outlined above. We can no longer assume radiative balance as in the previous section, as we have extra terms in that come into play from the convection. We will begin with the same radiative transfer equations

$$\frac{dU}{d\tau} = U - B \tag{16a}$$

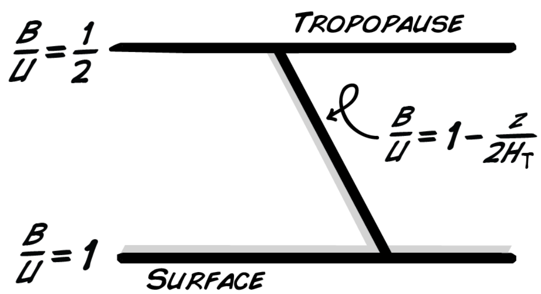

Figure 4: A depiction of the approximation used in the RCE model.

$$\frac{dB}{d\tau} = B - D. \tag{16b}$$

From here, we solve the equations in a different manner, and derive two equations with two unknowns: T_g, which still refers to the ground temperature and H_T, the height of the tropopause. The tropopause is the boundary between the troposphere and the stratosphere.

We will only consider the first of the radiative transfer equations, and rearrange to give

$$\frac{1}{U}\frac{dU}{d\tau} = 1 - \frac{B}{U}, \tag{17}$$

so that we can write it as

$$\frac{d(ln(U))}{d\tau} = 1 - \frac{B}{U}. \tag{18}$$

Next we make an assumption that has been shown to be a good match to data. As depicted in figure 4, we assume that the quantity B/U varies linearly from the tropopause down to the surface, according to the equation

$$\frac{B}{U} = 1 - \frac{z}{2H_T}, \tag{19}$$

where z is the height in the vertical direction (the altitude). This expression can be substituted into equation (18), which yields

$$\frac{d(ln(U))}{d\tau} = \frac{z}{2H_T}. \tag{20}$$

Since the right-hand side of the equation is now in terms of z, we make the substitution from τ to z in the derivative term, according to

$$\tau(z) = \tau_s e^{\frac{-z}{H_a}}. \tag{21}$$

Thus,

$$d\tau = \frac{-1}{H_a}\tau_s e^{\frac{-z}{H_a}} dz. \tag{22}$$

Note that here we use the symbol τ_s instead of τ_0 as in the RE case. This is because we use a different value of the optical depth when accounting for convection, in order to keep the ground temperature at 288K. The dependence on temperature still takes the same form, except for a scaling factor of 6.4 in the RCE case, compared to the RE constants. A higher value of the optical depth for convection than radiation makes sense, if we look at figure 3 and notice the rapid temperature increase very close to the ground in the radiative equilibrium temperature profile in blue. For convection to play its role and heat the ground to the same temperature, the optical depth must be greater.

The differential equation then becomes

$$\frac{d(ln(U))}{dz} = -\frac{z}{2H_T}\frac{\tau_s}{H_a}e^{\frac{-z}{H_a}}. \tag{23}$$

This equation can be solved using integration by parts, which gives

$$ln\left(\frac{U(z=H_T)}{U(z=0)}\right) = \int_0^{H_T} -\tau_s H_a e^{\frac{-z}{H_a}}\frac{1}{2H_T H_a}dz \tag{24}$$

$$ln\left(\frac{2\sigma T_T^4}{\sigma T_g^4}\right) = \frac{-\tau_s H_a}{2H_T}, \tag{25}$$

where T_T is the temperature of the tropopause, given by

$$T_T = \frac{T_e}{2^{\frac{1}{4}}}. \tag{26}$$

Before we can evaluate the left-hand side of the equation, we introduce an equation that will be used in the remainder of this section. The following equation relates the temperature of the tropopause to the temperature of the ground, which is the quantity we are truly interested in. To do this, we use the lapse rate (denoted Γ), which is a measure of how much the temperature in the atmosphere decreases with increasing altitude. i.e. $\frac{dT}{dz}$. For simplicity, we are assuming a constant lapse rate, and so the temperature of the tropopause is related to the temperature of the ground in a linear fashion:

$$T_T = -\Gamma H_T + T_g. \tag{27}$$

So Γ is the slope of the line in a temperature vs. height graph that connects the ground temperature with the tropopause temperature. In our model, we use a global average value of 6.5 K/km for the lapse rate [9].

Going back to equation (25), the left-hand side can be approximated as

$$ln\left(\frac{2T_T^4}{T_g^4}\right) = ln(2) + 4ln\left(\frac{1}{1+\frac{\Gamma H_T}{T_T}}\right) \tag{28}$$

$$= ln(2) - 4\left(\frac{\Gamma H_T}{T_T}\right), \tag{29}$$

where we plug in for T_g according to equation (27) and approximate the last term in the first equation above by using a Taylor series to obtain equation (29). If we put all of these approximations together, we get a quadratic equation in H_T:

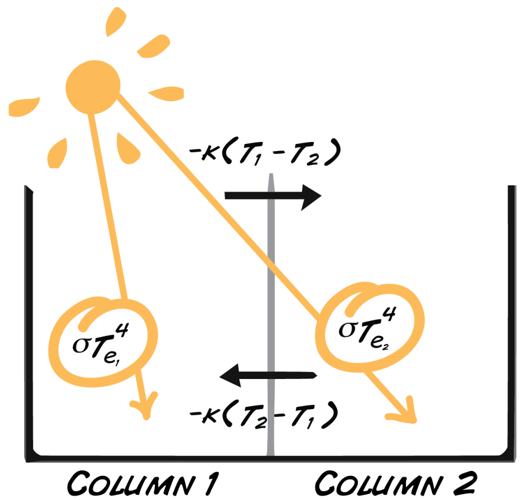

Figure 5: The two-column model of radiative balance with an absorbing atmosphere.

$$-8\Gamma H_T^2 + 2ln(2)(H_T T_T) + \tau_s H_a T_T = 0. \tag{30}$$

Solving for H_T gives one of the two equations used in the one-column RCE model, and the other is simply equation (25):

$$H_T = \frac{1}{16\Gamma}\left(CT_T + \sqrt{C^2 T_T^2 + 32\Gamma\tau_s H_a T_T}\right) \tag{31a}$$

$$T_g = T_T + \Gamma H_T, \tag{31b}$$

with $C = 2ln(2)$. Equations 31 are the two equations with two unknowns, the height of the tropopause H_T and the temperature of the ground T_g, that we solve to find the radiative-convective equilibrium solutions in a one-column model.

3.3 Two-Column Radiative Equilibrium

Now we take a step backwards and disregard convection, but create a two-column model that allows for lateral heat transport between the two columns. The setup is identical to the one-column RE model (see figure 5), with different values of the emitting temperature (T_e) and ground temperature (T_g), and thus also the optical depth (τ), as well as an added term defined as $k(T_{g_i} - T_{g_j})$ to represent the exchange of heat between the columns (where i and j denote either column 1 or column 2). Here, k is considered to be a diffusion coefficient, and this diffusion term is an approximation to the standard diffusion term $k\frac{d^2T}{dy^2}$, where y denotes the latitude. We consider column 1 to be a rough analogy to the tropics, and column 2 to represent the midlatitudes. This is taken into account by allowing for the emitting temperatures to be those corresponding to the specific regions.

The final equations for the two-column case are simply extensions from the one-column case, and therefore I will not go through the derivations a second time:

$$\sigma T_{g_1}^4 = (\sigma T_{e_1}^4 - k(T_{g_1} - T_{g_2}))(1 + \frac{\tau(T_{g_1})}{2}) \tag{32a}$$

$$\sigma T_{g_2}^4 = (\sigma T_{e_2}^4 - k(T_{g_2} - T_{g_1}))(1 + \frac{1 + \tau(T_{g_2})}{2}). \tag{32b}$$

As before, the optical depth τ is written as a function of temperature according to the equations

$$\tau(T_{g_1}) = a + b\left(e^{c\left(\frac{1}{288} - \frac{1}{T_{g_1}}\right)}\right) \tag{33a}$$

$$\tau(T_{g_2}) = a + b\left(e^{c\left(\frac{1}{288} - \frac{1}{T_{g_2}}\right)}\right). \tag{33b}$$

4 Preliminary Results

4.1 One-Column RE

We begin by showing the solutions for the ground temperature T_g of the RE equations (equations 14) as a function of the emitting temperature T_e, as shown in figure 6. There are a couple features worth noting; first, the shape of the solution resembles a backward C-curve. This implies that for temperatures below about 268K, there are two equilibrium solutions. The bottom branch is the branch we are currently on (for our current emitting temperature of 255K, the ground temperature is the expected 288K), and the top branch shows a much higher ground temperature for a given emitting temperature.

Another important feature is that there is a critical temperature (in this graph at 268K) above which there are no solutions to the equations. That is, there are no equilibrium points in the system, and we thus deduce that the system is in a runaway greenhouse regime, where the planet is endlessly warming.

We are also interested in the stability of these equilibrium points. If we start by considering a time-dependent equation:

$$C\frac{dT_g}{dt} = (1 + \frac{\tau}{2})\sigma T_e^4 - \sigma T_g^4, \tag{34}$$

and linearizing, we find that the bottom branch (the branch we are currently on) is stable, whereas the top branch is unstable.

In the one-column RE model, we also added in a term that allowed for additional heat loss (see equation 15), and the solutions in this case are shown in figure 7. Allowing for additional heat loss creates an added upper branch to the C-curve diagram, but one that curves upward rather steeply. This branch arches into an extremely high ground temperature range, and so is likely not of great importance to our current Earth (thankfully), but this does reveal an interesting aspect of the system that we are studying. If we take another look at equation 15, we notice that for high ground temperature, the kT term will dominate. The different shape of the curve, then, at high ground temperatures is not a surprise.

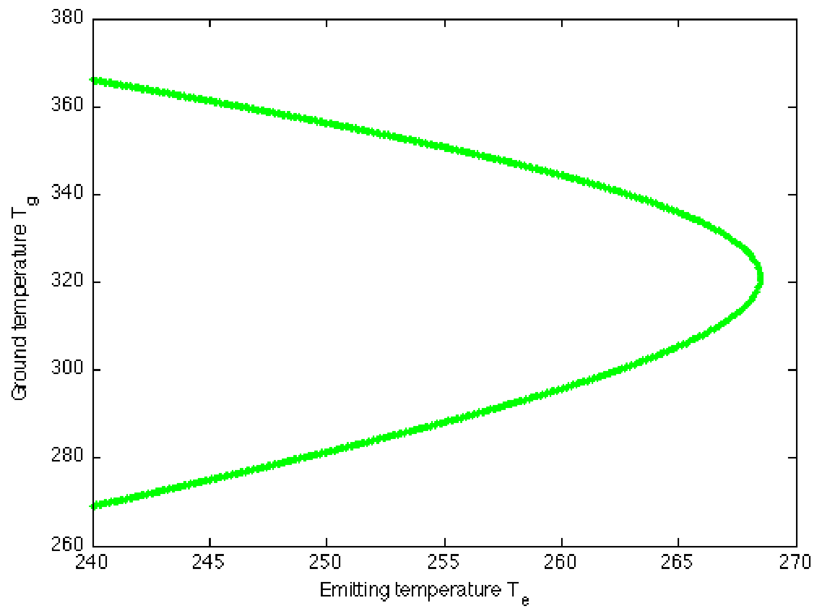

Figure 6: Emitting temperature plotted vs. ground temperature for the RE model.

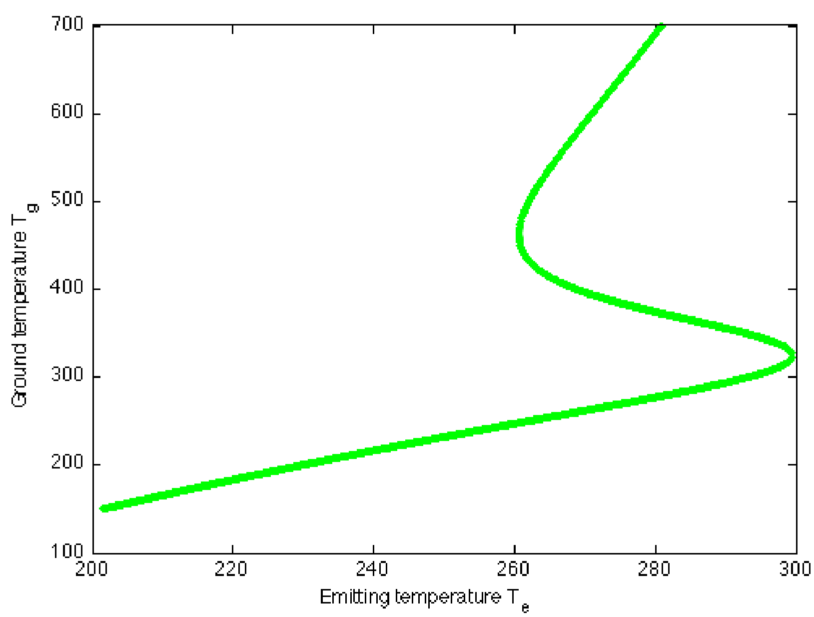

Figure 7: Emitting temperature plotted vs. ground temperature for the RE model with additional heat loss term.

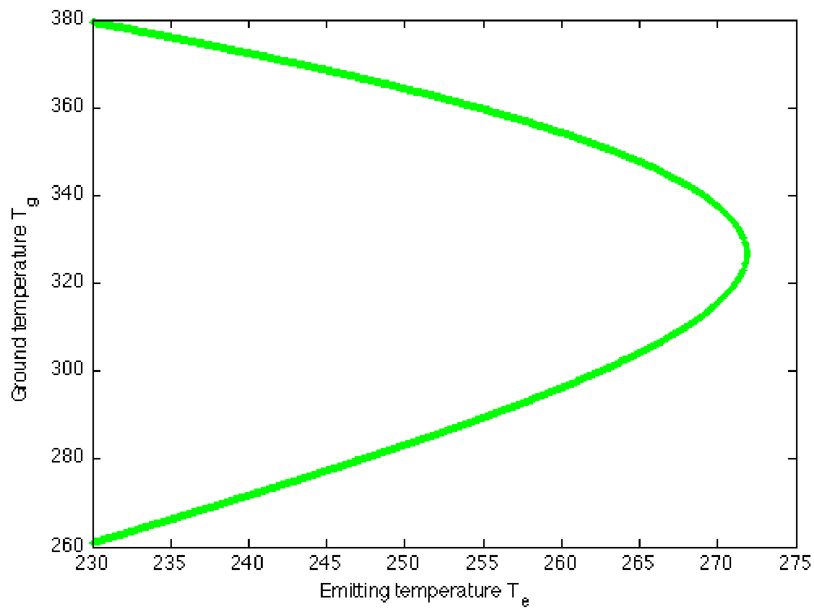

Figure 8: Emitting temperature plotted vs. ground temperature for the RCE model.

It is also worth noting that the tip of the C-curve is at a much higher critical temperature of nearly 300K. This also makes intuitive sense, as the extra term is allowing for heat to be taken out of the system, allowing for an equilibrium solution for greater amounts of incoming solar radiation.

4.2 One-Column RCE

In the RCE model, we observe the same C-shaped curve (see figure 8) as we did in the RE case. This time, however, the critical emitting temperature before the onset of a runaway greenhouse effect is a couple of degrees higher, around 272K. This is a bit misleading, however, because we have increased the constants in the equation for optical depth, and so the comparison must take this into account. In figure 9, we show the RE C-curve using the values for the optical depth in the RCE model. We see here that the critical emitting temperature has dropped to under 200K. When taking this into account, we see that adding convection to the model greatly mitigates the runaway greenhouse effect.

Since we had a second variable, H_T in the RCE model, we show the solution of the tropopause height also for varying emitting temperatures (see figure 10). The shape of the curve is still C-like, though slightly asymmetric. Still, there exist two solutions for a range of emitting temperatures, and above a critical value there exist no solutions.

This work is meant to be a study of the system described in previous sections, and so we present here a couple of parameter studies, meant to shed light on the behavior of our model. Figure 11 shows the model's dependence on the lapse rate Γ. The three curves plotted are the full solution for the ground temperature for 3 different values of the lapse rate, shown in the plot's legend. Similarly, figure 12 depicts the height of the tropopause

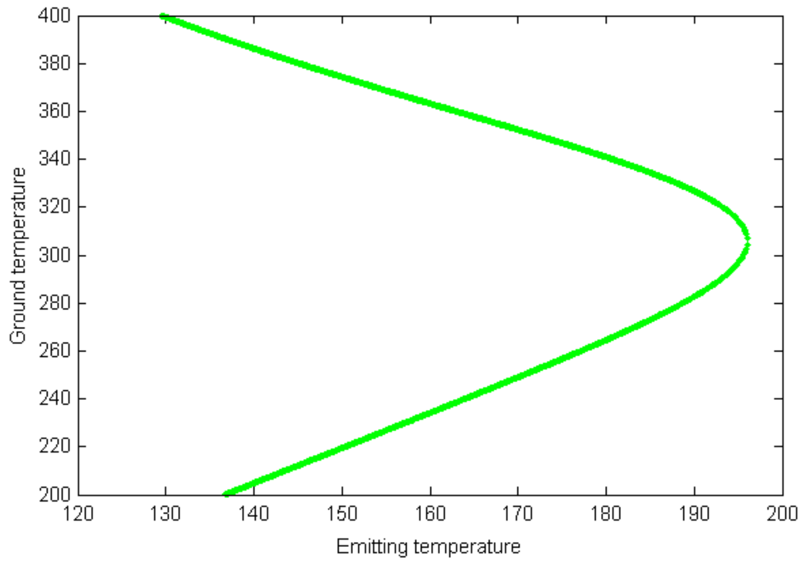

Figure 9: Emitting temperature plotted vs. ground temperature for the RE model when using the values for the RCE optical depth.

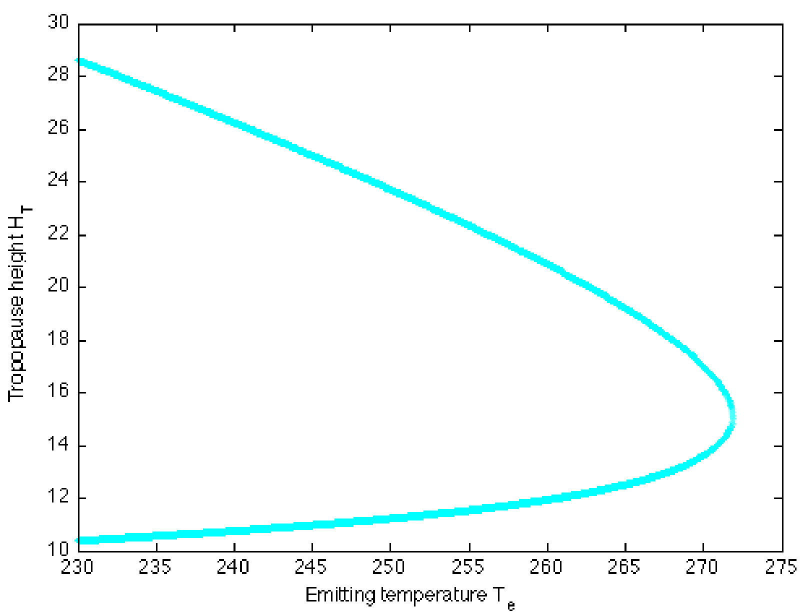

Figure 10: Emitting temperature plotted vs. ground temperature for the RCE model.

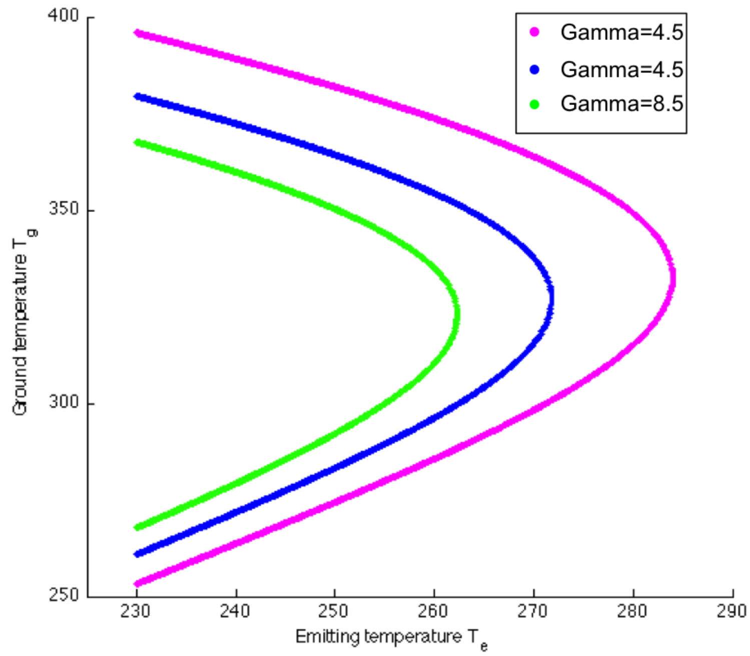

Figure 11: Emitting temperature plotted vs. ground temperature for the RCE model with three values of gamma.

for the same three values of the lapse rate. In both cases, as the lapse rate decreases, the critical emitting temperature increases and the range of ground temperatures increases, as well. In the previous plots, the lapse rate used was a constant 6.5 K/km. However, in reality the lapse rate also depends on temperature, and taking this into account would be a logical next step in this study.

Figure 13 illustrates the relationship between the emitting temperature and the lapse rate. As mentioned in the previous paragraph that the critical temperature increases with decreasing lapse rate, this figure shows the whole spectrum. According to the plot, today's emitting temperature of 255K would be critical if the lapse rate were a constant value of 10 K/km across the globe. We portray the same information in a different way in figure 14. The vertical axis still shows the lapse rate, but the horizontal axis shows the fractional distance to the sun (according to the Earth-sun distance) needed to obtain the critical emitting temperature. For example, the Earth is located at 1 Earth-sun distance and would reach critical emitting temperature at a lapse rate of 10 K/km. Venus, located at about 0.72 Earth-sun distance away from the sun requires a lapse rate of just over 2 K/km to achieve a runaway greenhouse effect. For the global average lapse rate of around 6.5 K/km [9], as used in this work, the Earth would need to be at roughly 0.9 Earth-sun distance to be at a critical emitting temperature, assuming all other elements of the system remain the same.

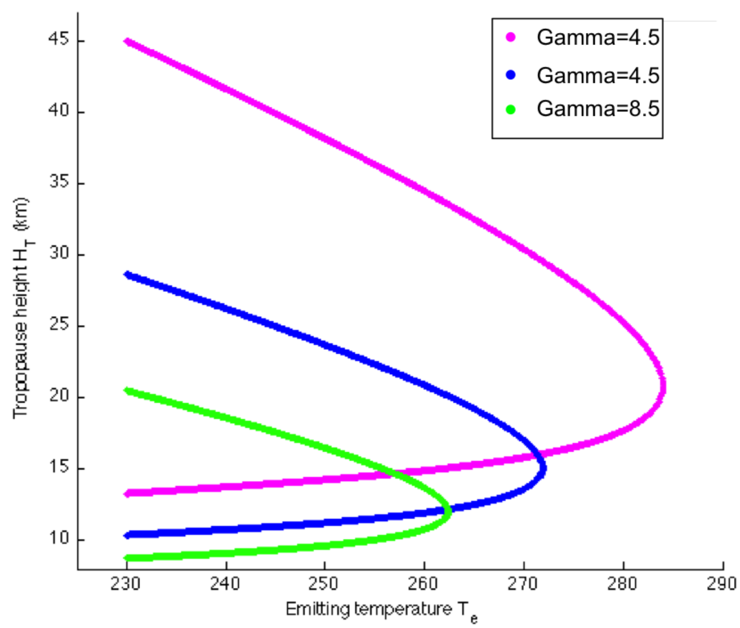

Figure 12: Emitting temperature plotted vs. tropopause height for the RCE model with three values of gamma.

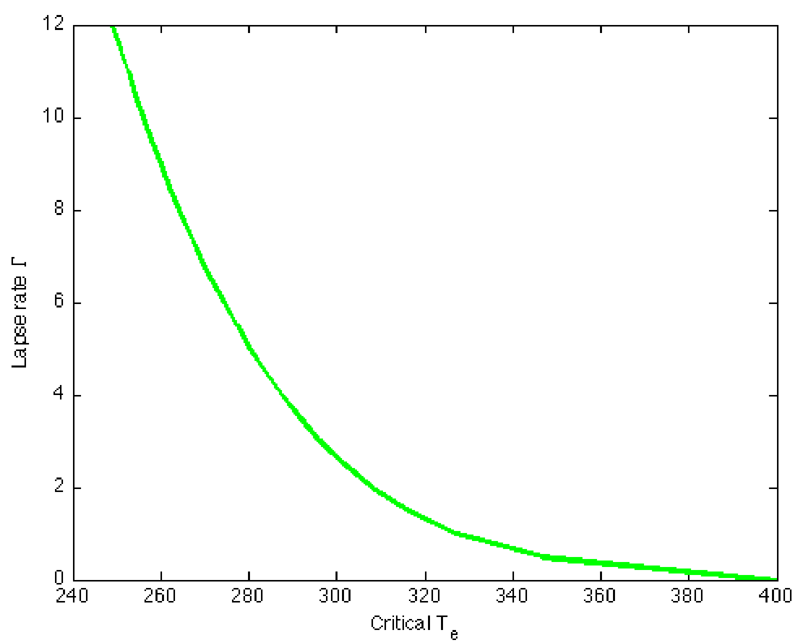

Figure 13: Critical emitting temperature plotted vs. lapse rate for the RCE model.

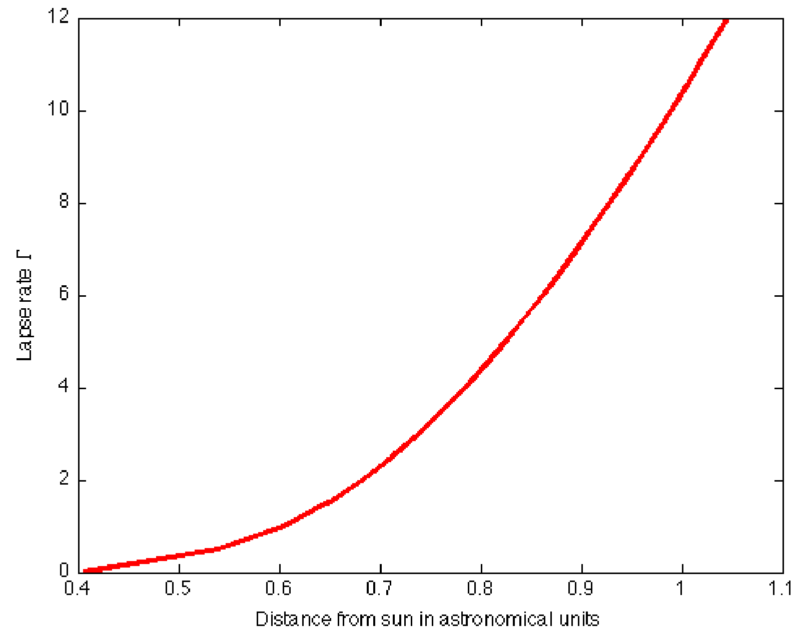

Figure 14: Distance to the sun vs. lapse rate for the RCE model.

4.3 Two-Column RE

Finally, we take a look at the two-column radiative equilibrium case. Figure 15 shows two plots, one for each of the two columns. Once again, we see the familiar C-curves, though this time there appear to be more than one C-curve in each column. Even though the plots do not show two full C-shaped curves, they both indicate that the full solution may actually be made up of two C-curves, as we postulated. This is more obvious in the top graph, for column 1 (the warmer temperature, mimicking the tropics). The outer C-curve is visible, and the upper branch of what appears to be a second C-curve is visible. We postulate that there is a missing lower branch to the inner C-curve, which may be missing due to numerical difficulties. Having two C-curves makes intuitive sense, since each column displays a C-curve separately as we showed in the one-column RE case, and with the exchange of heat between the columns, each column now portrays two C-curves in the solution. More work is certainly required to confirm our speculations.

Regarding the runaway greenhouse effect and its onset, the two-column model does not show a great improvement; i.e. the critical emitting temperature is only slightly higher than the 268K that we obtained from the single column case. The first column has a very similar value of around 269K, while the second column appears to have a slightly lower critical emitting temperature, which we infer to be in the neighborhood of 265K. However, the system is quite different with two columns, and the emitting temperature T_e is defined as 20 Kelvin lower in the second column than in the first. So it is possible that the two-column RE approach mitigates the runaway greenhouse effect very slightly, but the system is different enough and the numbers are not robust enough at this time to draw any further

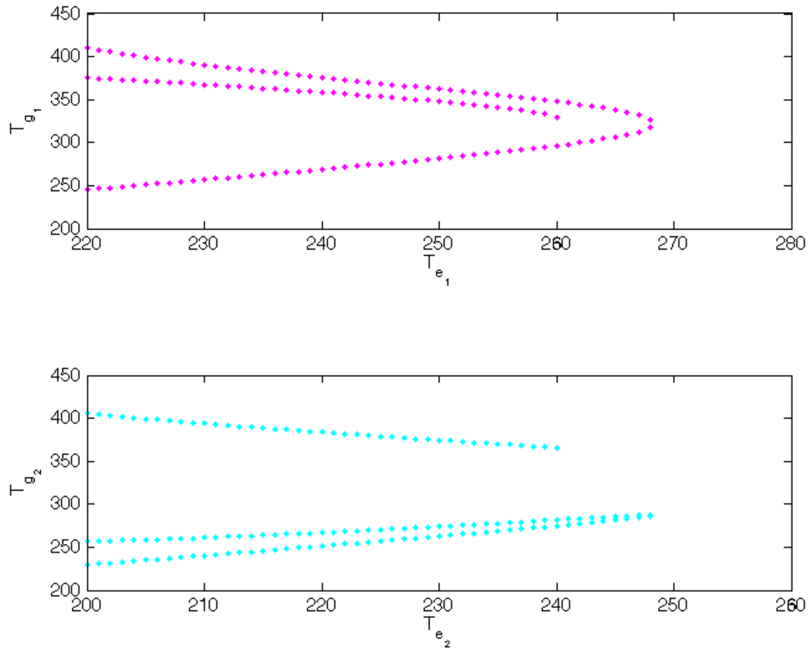

Figure 15: Emitting temperature plotted vs. ground temperature for both columns in the two-column RE model. Upper graph: column 1 (the tropics); Lower graph: column 2 (the midlatitudes).

conclusions.

5 Conclusions and Future Work

These results are preliminary, as this is but the starting framework for understanding the Earth system's behavior regarding the runaway greenhouse effect and lateral heat transport. However, there are a couple key features that are worth pointing out. We find that in both the RE and RCE models, there exists a critical emitting temperature, above which there are no equilibrium solutions, and the system experiences a runaway greenhouse effect.

In the one-column RCE model, we find a critical emitting temperature of around 268K. In the one-column RCE model, it is a couple degrees higher, around 272K. However, because we use a different value of the optical depth for the RCE than the RE model, these numbers are not easily comparable. In fact, if using the more realistic RCE value of the optical depth, we see that our planet would already be in a runaway greenhouse regime in the RE model. So, convection mitigates the runaway greenhouse effect quite a bit, raising the critical emitting temperature from just under 200K in the RE model to 272K in the RCE model.

Adding a second column hardly alters the critical emitting temperature, contrary to what we expected. With a warmer column pumping heat into a cooler column, we anticipated a higher critical emitting temperature in the first column, and thus a higher threshold for the onset of the runaway greenhouse effect. However, the two-column model is still in need of tuning and we will thus continue developing the system and hopefully better understand it in a future work.

Beyond a more in-depth analysis of the two-column RE model in the future, we would like to add an analysis of a two-column RCE model as well. We would also like to express the lapse rate as a function of temperature, in addition to the temperature-dependence of the optical depth, as we have done here. Finally, we would like to perform more parameter studies in order to have a more thorough understanding of the Earth system's behavior.

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