GFD Project: Exploiting Self-Organized Criticality in Strongly Stratified Flows

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Strongly stratified turbulence is a fundamental agency for diabatic mixing in numerous geophysical flows [1]. Nevertheless, parameterization of the mixing efficiency in stratified turbulence remains a subject of debate, particularly in physically-relevant parameter regimes in which the Reynolds number $Re \to \infty$ while the Froude number (an inverse measure of the stratification strength) $Fr \to 0$. In this extreme parameter regime, the flow is dominated by highly anisotropic structures with horizontal scales much larger than their vertical scales. Owing to their relative horizontal motion, these structures are susceptible to stratified shear (e.g. Kelvin–Helmholtz and Holmboe) instabilities that drive spectrally non-local energy transfers. Collectively, these attributes make both DNS and LES of stratified turbulence especially challenging.

To surmount these difficulties, we have performed a multiple-scale asymptotic analysis [2] of the Boussinesq equations in the dual limit $Fr \to 0$ and $Re \to \infty$. The resulting *generalized quasi-linear* (GQL) model [3] captures the essential physics of strongly stratified turbulent shear flows: the slowly-evolving mean fields are governed by the hydrostatic primitive equations augmented with the vertical divergence of Reynolds stresses and buoyancy fluxes arising from the isotropic and non-hydrostatic fluctuation dynamics.

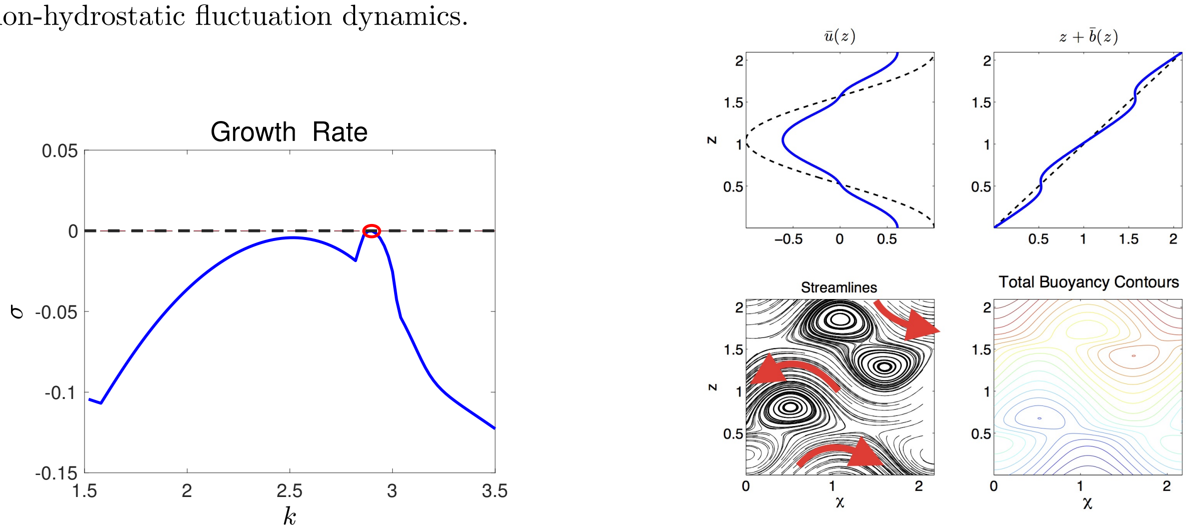

Figure 1: Exact coherent state in strongly stratified Kolmogorov flow at $Fr = 0.01$, $Re = 10^5$ supported by a single *marginally-stable* horizontal mode with wavenumber $k = 2.9$.

The model has been used to investigate the mixing efficiency of certain exact coherent states (ECS) arising in strongly stratified Kolmogorov flow (see figure 1). The ECS are computed using a new methodology for numerically integrating multiple time-scale QL systems [4] that obviates the need to explicitly resolve the fast dynamics associated with the stratified shear instabilities. The key idea is that the slowly-evolving amplitude of the fluctuations must be instantaneously slaved to the slow mean fields to prevent positive fluctuation growth rates from being realized; that is, the slow dynamics is constrained to a *marginal stability* manifold, a mathematical expression of self-organized criticality. The aim of this GFD project is to extend this new formalism to incorporate slow horizontal spatial variability of the mean fields, first in the context of a toy PDE and then for stratified flow, and to allow for the inclusion of multiple marginal modes. An important physical application is to the saturation of the zig-zag instability via non-local energy transfer to KH modes.

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[2] K. Julien & E. Knobloch (2007), *J. Math. Physics* **48**: 065405.
[3] B. Marston, G. P. Chini & S. Tobias (2016), *Phys. Rev. Letters* **116**: 214501.
[4] G. Michel & G. P. Chini (2019), *Proc. Roy. Soc. A* **475**, Issue 2223.