GFD Project: Langmuir Circulation and Cellular Contour Dynamics

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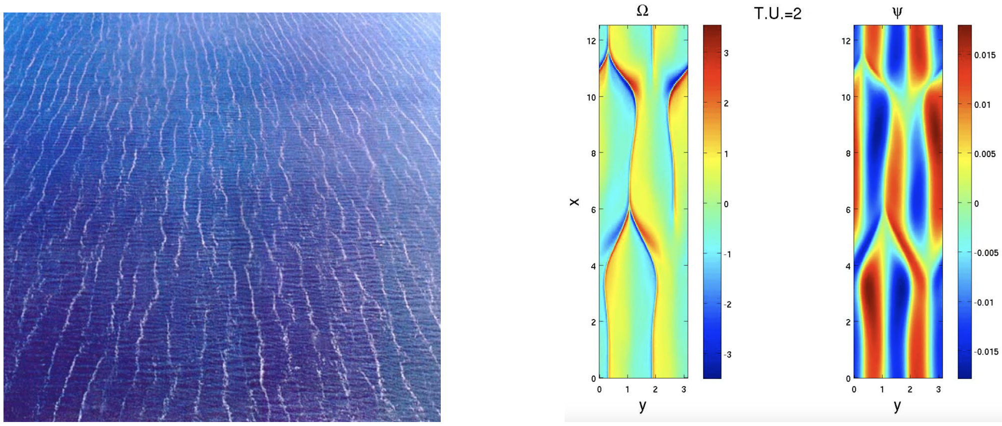

Figure 1: Langmuir circulation surface signatures: windrows on the Great Salt Lake (left), and downwind (x) vorticity Ω and cross-wind (y,z) streamfunction ψ from a simulation of the Craik–Leibovich equations (right). Note that Ω is piecewise uniform, while ψ is smooth.

Langmuir circulation (LC) is a wind- and surface-wave-driven flow in the ocean surface mixed layer comprising an array of counter-rotating roll vortices (or 'cells') having axes aligned with the wind and waves. Theoretical and numerical work examining the quasi-steady response of a homogeneous layer to wind forcing in the presence of surface-wave Stokes drift has shown that the downwind vorticity is homogenized within each cellular patch, per the Prandtl–Batchelor theorem. The patches of constant vorticity are separated by plume-like structures carrying vertical fluxes of high- and low-speed streamwise momentum. A natural and open question concerns the long-time spatiotemporal evolution of these cellular and plume structures.

This question prompts two related modeling approaches, which could be pursued separately, sequentially, or in parallel. Firstly, a preliminary strategy would be to neglect the *direct* dynamical effects of surface waves and consider cellular-patch solutions of the 2D Euler equations in a periodic strip with horizontal period L and depth D. In this scenario, a flow consisting of piecewise constant regions of vorticity can be examined solely by tracking the evolution of the boundaries between the cells. Possible research questions, approaches, and tasks for this project include: (i) What are the steady states with vertical cell boundaries (e.g. one likely solution is symmetric with vorticity $\pm q$ in two equal-length regions)? Are they stable? (2) Develop a *contour dynamics* algorithm to compute the flow. (iii) Add a lower, possibly semi-infinite layer, initially at rest. Flow will be induced by the horizontal velocity at the base of the mixed layer. How much energy is lost? To examine coupling with internal waves, the lower layer could have a different density or be uniformly stratified.

The second modeling approach would directly account for the phase-averaged effects of surface waves through their Stokes drift using the so-called *Craik–Leibovich* (CL) equations. The project would involve generalizing the analysis of Chini in *J. Fluid Mech.* (2008), vol. 614, pp. 39–65, to account for slow spatiotemporal evolution of the cellular patches. In this scenario, the cells are not governed by the Euler equations, so existing counter dynamics formalisms cannot be applied. New mathematical, modeling, and algorithmic approaches are required that exploit features of the quasi-steady cellular solutions to enable model reduction (in particular, the relative smooth spatial variation of the streamfunction). Can a reduction to a surface restriction of the CL equations be derived? As in the first scenario, extensions could include incorporating a stratified lower layer, so that LC–internal-wave coupling could be investigated.