Optimal Dynamos

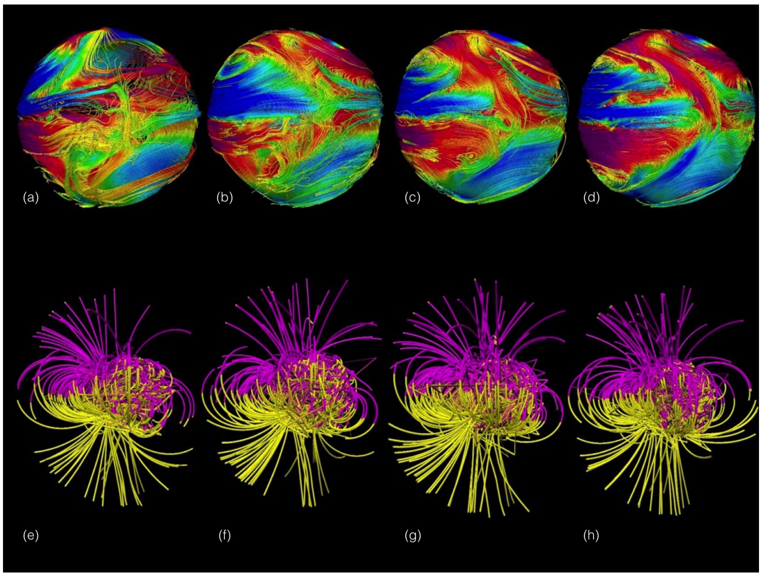
Jim McElwaine

The flow of conducting fluid can gives rise to dynamos which are responsible for the magnetic field of many planets, including Earth, and play a crucial part in the dynamics of stars and some exo-planets. In exo-planets in particular the dynamo may be maintained by conductivity variations arising from strong asymmetric heating from the planets' host star. The presence of a dynamo significantly increases the surface magnetic field strength and alters the overall planetary magnetic field geometry, possibly affecting star-planet magnetic interactions. Dynamos driven by conductivity variations have been little studied and 2d dynamos may even be possible as the standard proof of their non-existence breaks down if the conductivity varies.

The magnetic induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} - (\nabla \eta) \times (\nabla \times \mathbf{B}), \tag{1}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{v}$ the velocity field and η is the magnetic diffusivity (inverse conductivity). Although the dynamo mechanism by conductivity variations is subtle, one can show that given the correct alignment between $\nabla \times \mathbf{B}$ and $\nabla \eta$ the last term on the right hand side of Equation 1 can provide a positive α effect, thus regenerating poloidal field from toroidal and closing the dynamo loop. The aim of this project is to consider what velocity fields $\mathbf{v}$ and magnetic diffusivity fields η give rise to the fastest growing dynamos using techniques from homogenisation theory.



Time snapshots of toroidal magnetic field (a–d)
and the radial magnetic field (e–h).

References

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