Spatial Heterogeneity and its Relation to Processes in the Upper Ocean

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February 19, 2004

Abstract

In the ocean, the spatial distribution of biogeochemical tracers is affected by their physical transport in the fluid medium. Many tracer distributions such as sea surface chlorophyll and temperature, are highly correlated at length scales of 1–100 km on account of a commonality in the transport processes that affect them. We characterize and differentiate between the spatial heterogeneity of the tracers by using a variance-based measure for "patchiness". When we analyze the satellite-derived fields of surface chlorophyll and temperature, we find that chlorophyll is more patchy than temperature, i.e., a greater proportion of its variance occurs at small scales. We explain such differences in heterogeneity by taking the approach that the observed spatial heterogeneity of a tracer results from a balance between processes that generate variance and those that shift the variance from one length scale to another. The longevity of the tracer determines the extent to which the variance can be shifted to another scale. In the surface ocean, variance introduced at large-scales due to geographic variations, can be driven to smaller scales by the horizontal stirring and stretching of fluid filaments. On the other hand, small-scale vertical motion associated with fronts introduces small-scale variance that spreads to larger scales if the tracer anomalies are long lasting. For the latter case, we derive a quantitative relationship between a tracer's patchiness, and the time scales of processes that modify its concentration in the upper ocean. This relationship links the observed spatial heterogeneity in the system to the processes that contribute to its generation. It lends hope to our being able to use quantitative measures of spatial heterogeneity, like the patchiness parameter defined here, to gain information about processes or vice versa, to predict how the spatial heterogeneity might be modified as a result of a change in processes.

Introduction

A key factor that influences spatial heterogeneity in the ocean and distinguishes it from heterogeneity in landscapes is that substances or properties in the ocean are transported within the fluid medium, which is in motion. Hence the variability in the distribution of properties is largely linked to the dynamics of the fluid, which is complex since it varies in both space and time. Spatial heterogeneity in the ocean is constantly evolving in time, in contrast to terrestrial systems, where the variability of the underlying medium (for example, the geology or soil conditions) is more or less static on the time scales of relevance in the ocean. The fluid dynamical processes act over a wide range of time and length scales. In addition, there are a number of processes like warming or cooling at the surface, evaporation and precipitation, biological production of phytoplankton, and air-sea gas exchange, that alter the properties of the ocean. Some processes

generate variability and others annihilate it; our objective is to understand the net effect of these factors on the distribution of properties.

Transport in the ocean occurs through the process of advection, that carries properties along with the flow, and diffusion, due to which substances or energy can spread through the fluid. Diffusion is generally associated with small scales; it is important to individual plankton and bacteria and they may rely, for example, on the a spatial variation in the concentration of a nutrient for its transport. A net diffusive flux occurs without the input of energy when the concentration gradient of a substance is spatially varying. The molecular diffusivity κ, of most substances is small and thus diffusive transport (quantified as $\kappa\nabla^2 c$ where c is the tracer concentration and ∇ the gradient operator) is relevant only at small length scales (the diffusive length scale $L_{\text{diff}} \sim \sqrt{\kappa T} \approx 1$ mm, for $\kappa = 10^{-5}$ ms^{-2} and a time interval $T = 10$ s). At longer times T, and at length scales greater than a centimeter or so in the upper ocean where typical velocities U are in the range of 0.01 to 1 m s^{-1}, advective transport by the fluid by far dominates diffusion. A net advective flux of tracer, $\mathbf{u} \cdot \nabla c$, occurs when there is a concentration gradient in the tracer ∇c, in the direction of the fluid velocity $\mathbf{u}$. The length scale associated with advection, $L_{\text{advec}} \sim UT$, increases linearly with time T, as compared to the 1/2 power in the case of diffusion. Hence with increasing time, advection affects larger length scales than diffusion. A process of considerable relevance in the ocean, and somewhat different from pure advection and diffusion, is mixing. It transfers energy and property gradients from larger to smaller scales and results in the homogenization of properties over the region on which it operates on relatively short time scales. Mixing, which is often induced by shear and convective instability, contributes much more to the flux of energy and tracers than molecular diffusivity, which is relatively negligible for length scales of more than a few centimeters. The effective flux of energy or tracer generated by random turbulent motions is often parameterized as a diffusion-like process by using an enhanced "eddy" diffusivity κ_{eddy}.

Another aspect that differentiates the ocean from the land surface is that it is a three-dimensional medium. However, the rotation of the earth, the small geometrical aspect ratio of depth to length scales in the ocean, and density stratification, all contribute to the fact that motion in the horizontal plane dominates vertical motion. In fact, vertical velocities are several orders of magnitude smaller than horizontal velocities when one considers length scales of the order of tens of meters or more. Hence, even though the ocean is three-dimensional, it is highly layered. The variation (i.e. the gradient) of properties is much stronger in the vertical than horizontal, but motion is much more rapid in the horizontal. Hence the distributions of properties evolve more rapidly in the horizontal plane. The upper ocean, in particular, is more energetic and fast-moving than the deep. Hence the property distributions are rife with a highly transient variability that is influenced strongly by advection and mixing within the fluid, as well as forcing factors the modify the properties. Figure 1 displays the kind of spatial variability that results from the coupling between the biological and fluid transport processes.

In this chapter, we focus largely on upper ocean heterogeneity that has a transience time scale of the order of weeks and a coherence length scale in the range of 1–100 km. Since the fluid dynamical transport processes are common to the various substances or properties in the ocean, it it no surprise that the distributions of different tracers in the ocean are correlated on these length and time scales. Yet, the spatial heterogeneity of one property can vary from that of another to which it is closely linked. One challenge is therefore to quantitatively relate the heterogeneity of properties to the processes that affect them. Such an understanding might enable us to use one property as a proxy for the distribution of another, and secondly, learn something about the processes at play from the characteristics of the distribution. Further, we might be able to anticipate a change in spatial heterogeneity resulting from a change in the processes or controlling parameters.

In general, the spatial and temporal heterogeneity of a system depends on the length and time scales of relevance and the process under consideration. Questions concerning the importance of heterogeneity, how we may model it, and how it might affect the ecosystem function are difficult to generalize and best viewed in a specific context. But, a somewhat general set of questions

about heterogeneity and its link to processes that I would like to consider are:

1. What role does heterogeneity play for the question at hand? What is the effect of varying the heterogeneity (or some measure of it) on an ecosystem function, an ecosystem response, or on an integrated measure of interest?
2. How might one quantify the heterogeneity of the system?
3. What are the most important parameters/ factors/ processes behind the heterogeneity? How does the heterogeneity vary as a function of these parameters?
4. What is the relationship between the heterogeneity, the ecosystem response and the parameters on which the heterogeneity depends?
5. Is it possible to account for the heterogeneity without explicitly modeling or measuring it? How might one sample a variable to correctly estimate an integrated measure of it?

Discussed below, are some thoughts relating to these questions. Following this, I describe a study that attempts to explain and relate the spatial heterogeneity of different tracers at the sea surface over length scales in the range of 1–500 km.

The Relevance of Heterogeneity

Most oceanic processes are intermittent in space and time and it is often important to account for variability to gain an accurate estimate of a process or flux. This is particularly true when a processes in non-linear, in which case, even an integrated flux measure across the system's boundary requires a description of the heterogeneity. Let us consider, for example, the flux of carbon dioxide (CO_2) across the air-sea interface. We would like a time-integrated estimate of the flux of CO_2 into or out of the ocean, though the flux varies in time depending on the properties of the sea water, wind and surface conditions. The air-sea gas flux is generally parameterized as the product of a gas exchange coefficient k, the solubility of the gas, and the difference in the partial pressure of the gas ΔpCO_2, between sea and air. The gas exchange coefficient k is estimated empirically and is typically a function of the wind speed raised to a power that varies between 1.6 and 3, depending on the formulation [*Liss & Merlivat* (1986); *Wanninkhof* (1992); *Wanninkhof & McGillis* (1999)]. This non-linear dependence implies that short bursts and gusts of wind are more effective in fluxing CO_2 across the air-sea interface than a constant wind of the same mean intensity. Since the average of the instantaneous wind speed when squared, is not the same as the square of the wind speed averaged in time, the averaging period and the frequency of sampling the wind speed become relevant to the estimate of air-sea gas transfer that one would obtain from such a relationship. An estimate for the global air-sea flux of CO_2 can vary by a factor of two depending on whether we use monthly-averaged or 6-hourly winds to compute the fluxes. Any covariance between the variables k, s and ΔpCO_2 also affects the estimate and requires accounting for each of their variabilities independently over short time scales. Thus, the required resolution or the permissible period of averaging that is required to capture a process is highly dependent on the process and the distributions of the variables themselves. In this case, the integrated flux in and out of the system is dependent on the heterogeneity at the boundary since the process has non-linear dependencies.

As another example, consider the new production rate of phytoplankton in the subtropical gyres of oceans. New production (as opposed to the production that feeds off recently recycled organic matter) is derived from the supply of fresh nutrients from the subsurface, a processes that is highly episodic in time and space. A snapshot view of the ocean does not adequately represent this process, but the time-integrated effect of the process affects its state. Though the transport of nutrient by fluid advection may be considered a linear process, it is dependent on the spatial gradients in the nutrient. Quicker uptake of nutrient in the upper ocean and more efficient lateral transport at the surface, ensures a steeper vertical gradient in the nutrinet concentration

and a greater net flux to the surface from below. Patchy upwelling generates a heterogeneous surface distribution of nutrient. This enhances the lateral nutrient transport and leads to stronger vertical nutrient gradients at the upwelling sites, consequently resulting in a greater supply of nutrients from the subsurface as compared to the situation where the upwelling is uniformly distributed in space [*Martin et al.* (2002)]. In this case the transport of nutrient within various components of the system are linear processes, but the net productivity is nonetheless affected by the heterogeneity in the processes and distributions within a system. The net productivity in turn, affects the surface distribution of pCO_2 and the flux of CO_2 in and out of the system.

On much smaller scales, the transport of oxygen to a patch of decaying organic matter is dependent on the spatial heterogeneity in the oxygen distribution since the diffusive flux is proportional to the second spatial derivative of the concentration. If the supply of oxygen is rate-limiting to the process, then the heterogeneity of the oxygen distribution which may be generated by the bacterial uptake itself, is crucial for this activity. In such a case, the "patch" of decaying matter is not self-contained and depends on the spatial heterogeneity generated by itself or its neighbors for its survival. The turbulent diffusion of mechanical energy also occurs on a similar length scale to that at which diffusion operates because the molecular diffusivity of momentum is comparable to that of a trace substance in the fluid. However, turbulent dissipation is itself intermittent and the intermittency in the turbulent energy dissipation rate is found to account for a decrease in zooplankton-phytoplankton encounter rates by 25–50%, an increase in the nitrogen flux to non-motile phytoplankton cells by 6–62%, and a decrease in the coagulation and sedimentation of phytoplankton cells by 25–40% in experiments [*Seuront* (2001)].

What causes Spatial Heterogeneity?

One way of thinking of spatial heterogeneity is that it results from competing processes, one set that tends to homogenize the distribution of a property, and another, that tends to introduce variance or heterogeneity in the system. If one considers, for example, temperature in the ocean, it is homogenized by mixing and diffusion at small scales, but unequal heating or cooling generates spatial heterogeneity in its distribution on very large scales. At the intermediate scale, one could think of long wave radiation as relaxing the temperature to ambient atmospheric conditions, and advective motions in the fluid as generating heterogeneity by stirring. The observed spatial distribution would be more homogeneous if the diffusion-like mixing processes were relatively vigorous or the relaxation to an ambient state were more rapid, and more heterogeneous if the unequally heated regions were stirred into fine scale structures more rapidly than can be homogenized. Processes like diffusion or relaxation to an ambient state tend to generate uniformity in the fields, while specific sources for the properties, like biological reproduction or generation by nucleation, create heterogeneity. In the case of phytoplankton, heterogeneity is generated by the variable response of phytoplankton to varying physical properties and the availability of light and nutrients, but also by their reproduction which is dependent on the presence of mature phytoplankton cells. They are, however, removed or reduced to an ambient state of low concentration by predation, death and sinking. Their distributions are also homogenized by mixing and made more heterogeneous by advection which can generate narrow filamentous structures by stirring.

Relating Heterogeneity to Process Time Scales

The extent of the spatial heterogeneity in the distribution of a property results from the balance between the processes that homogenize and generate heterogeneity. These processes can be quantified in terms of the time scale on which they alter the concentration of the property. Hence, the rate of change of concentration c of a property can be expressed as

$$\frac{\partial c}{\partial t} = \frac{c}{\tau_V} - \frac{c}{\tau_H}, \tag{1}$$

where τ_V is a time scale on which variance is increased in the system and τ_H is a time scale on which the distribution is homogenized. In the case where c describes the concentration of phytoplankton whose heterogeneity is considered over length scales ranging from 0.1–1 m, τ_V

could the time scale of net growth or reproduction, while τ_H might be the time scale of diffusion $\sim L^2/\kappa$, where L is a length scale and κ, the kinematic or eddy diffusivity. In the statistically steady state, it is the balance between the right hand side terms in the above equation that determines the characteristics of the distribution. Hence it is the ratio of time scales τ_H/τ_V that determines the degree of spatial heterogeneity of the system. Later in this paper, we will show how the patchiness or spatial heterogeneity varies with the ratio τ_H/τ_V, when variance is introduced at the small scales.

Accounting for Heterogeneity

We have seen earlier, that the spatial heterogeneity of different properties can vary substantially. Thus, the grid resolution required in models and observation networks depends on the spatial heterogeneity of the property, given that one would like to observe the majority of its variance. Highly heterogeneous distributions require more resolution. Once again, the factor by which the resolution needs to be scaled up when going from one property to a more heterogeneous one, can be related to the ratio τ_H/τ_V for each of these. The question of how to account for heterogeneity is a more difficult one. It depends on the function or process that one wishes to account for (how it depends on the heterogeneous property), and also on the statistical characteristics of the property's distribution.

Quantifying Heterogeneity

Several methods have been used to quantify heterogeneity in the oceans. The most common among these are spectral analysis [*Platt & Denman* (1975); *Gower et al.* (1980)], semi-variogram analysis [*Yoder et al.* (1987); *Yoder et al.* (1993); *Glover et al.* (2002); *Deschamps et al.* (1981)] and autocorrelation analysis [*Campbell & Esaias* (1985)], probability density functions (pdf's), structure functions and multifractals [*Seuront et al.* (1999)]. In general, these methods analyze the variability of a distribution as a function of the length scale. A method that we have chosen to use in this presentation, characterizes the variance as a function of the size of the region. When a greater proportion of the variance lies at smaller length scales, we tend to refer to the distribution as more heterogeneous, patchy or intermittent. Once again, this depends on the range of length scales that one is considering.

In terms of processes, one may think of those that tend to shift the variance in a distribution to smaller scales, or others that obliterate (smear) it. In a fluid, advection or stirring, tends to drive variance to smaller scales because fluid filaments interleave and fold, generating finer scale filaments. Thus stirring two fluids generates one in which variance moves downscale with time. Hence the length scale at which the variance or heterogeneity is initially introduced, is relevant. If it be at the large scale, then variance can increase with time due to advection by the fluid. But if a process introduces heterogeneity at the small scale, then it gets annihilated with time due to processes like diffusion that smear gradients and reduce variance.

The Distribution of Biogeochemical Tracers at the Sea Surface

Sea surface temperature (SST) and chlorophyll (Chl) are two properties of the ocean that can be remotely measured from satellite platforms at a global resolution of approximately 1 km $\times$ 1 km. The distributions of these properties are highly correlated because both Chl and temperature are advected by the same underlying flow (Figure 2a). Oceanographic flow is largely two-dimensional; velocities in the vertical are several orders of magnitude smaller than in the horizontal when the length scales considered are such that the geometrical aspect (depth to length) ratio is small. Yet, these vertical velocities, which are on the order of tens of meters per day at most, can introduce anomalous concentrations at the surface because the concentration gradient of most substances and properties in the ocean is very large in the vertical as compared to the horizontal. Upper ocean processes, such as air-sea exchange, heating , evaporation and phytoplankton production modify the concentrations of substances, while the slow rate of mixing across the thermocline (which extends to a depth of 500 or so meters beneath the surface mixed layer and is a region over which

temperature and density change rapidly with depth) maintains a strong concentration gradient in the vertical. This common characteristic in the distribution of various properties ensures a similar response in their sea surface concentrations to upwelling, and consequently to horizontal advection that stirs the anomalous signatures introduced at the sea surface by upwelling.

In the following sections I describe a study in collaboration with J.W. Campbell [*Mahadevan & Campbell* (2002); *Mahadevan & Campbell* (2003)] in which we relate the spatial heterogeneity of tracers at the sea surface to the characteristic response time of processes that modify them in the upper ocean. Our interest is in quantifying and understanding processes such as the air-sea flux of CO_2, new production, the upwelling rate of nitrate (a key nutrient for phytoplankton), organic carbon export from the surface ocean and rates of remineralization of detrital organic matter, that are related to the carbon cycle in the ocean. Spatial heterogeneity seems to affect several of these processes and their rates; hence we would like to examine the reasons for the spatial heterogeneity in biogeochemical distributions and relate them to the underlying processes.

This study was motivated largely by the need to make a connection between remotely sensed variables like SST and Chl, and those that are more difficult to observe but play an essential role in the carbon cycle, such as the total dissolved inorganic carbon content (TCO_2) and oxygen (O_2). The questions of concern are: Why do the spatial heterogeneities of these substances differ and how may they be related? How may we account for this when modeling or observing different variables.

When we analyze satellite data for the sea surface distributions of Chl and temperature in the pelagic ocean, we find they are correlated, but Chl has a greater percentage of its variance at smaller length scales as compared to temperature. This analysis is done by computing the variance in two concurrent views of SST and Chl in a region covered by 256×256 pixels of data, where each pixel is approximately 1 km^2 in area. The variance corresponding to a length scale L is computed as $V(L) = N^{-1}\Sigma(x_i - \bar{x})^2$, where N is the number of pixels, x_i is the value of the variable at the i^{th} pixel, and $\bar{x}$ is the mean over all N pixels. L is the dimension of the box over which the variance is computed. Having computed the variance V_1 over the whole domain of size L_1, we divide the domain into boxes of consequently smaller sizes $L_1/2, L_1/4, L_1/8$...etc. and compute the average (over all the boxes in the domain) variance associated with each box of size $L_1/2, L_1/4, L_1/8$.... We normalize the variance $V(L)$ by the total variance in the domain V_1 so that we can compare the correspondence between variance and length scale for different variables. In Figure 2b, we show V plotted against L on log-log axes for Chl and SST from different regions of the ocean. There are two notable features in these plots: (a) The plots of $\log V$ *v.s.* $\log L$ are more or less linear, suggesting that

$$V \sim L^p \tag{2}$$

for Chl and SST over this range of length scales, and (b) p(Chl) is consistently less than p(SST). The slope p of the $\log V vs. \log L$ plots is a measure of the spatial heterogeneity of the distribution. Smaller p implies that a greater percentage of the total variance is contained in small scales and we think of the distribution as more "patchy". Hence p can be considered a patchiness index or a measure of the spatial heterogeneity, smaller p corresponding to greater patchiness. Sea surface Chl is found to be more patchy than SST for length scales ranging from 1-500 km. The reason for this is explored in the next few paragraphs where p is related to the properties of the tracer.

In an earlier study [*Mahadevan & Archer* (2000)], we examined the variability of different biogeochemical tracers within a model for a region of the subtropical gyres of the ocean. This model is initialized with nutrients (for phytoplankton) absent from the surface waters, but increasing with depth. When upwelling brings nutrients to the surface sunlit layers, they are converted to organic matter by the new production of phytoplankton. The phytoplankton production takes up TCO_2 in the surface layer, but TCO_2 is also modified by the air-sea exchange of CO_2 gas on a much longer time scale (several months to a year because CO_2 in the ocean equilibrates very slowly with the atmosphere). Dissolved O_2 is taken up by the remineralization of organic matter produced in the model, released by phytoplankton production and also subject to air-sea exchange, but with a shorter equilibration time than CO_2. We include two idealized tracers that

are initialized as varying exponentially from 0 at the surface to 1 at depth over an e-folding distance of 100 m. Their concentration changes due to the flow, but they are restored to their initial concentration profiles on time scales of 60 days and 3 days. The slowly restored tracer is meant to mimic dissolved organic carbon (DOC) and the fast-acting tracer resembles hydrogen peroxide H_2O_2.

When we analyze the surface distribution of the various tracers in the model, we find that the faster responding tracers or properties like new production, H_2O_2 and O_2 develop smaller scales or more patchiness than their respective counterparts: temperature, DOC and CO_2 (Figure 3). This dependence on the time scale can be understood by considering the balance between the processes that generate variance (at the small scale) and those that obliterate it or shift it to larger scales. The range of length scales considered here (the sub-meso and meso scales covering 1–500 km) is rather energetic. This is because the natural length scale of frontal instabilities and eddies in the ocean, the so-called the internal Rossby radius of deformation, lies within this range; it is typically between 10–100 km. Upwelling associated with fronts and eddies occurs along the edges of frontal meanders at sub-mesoscales (1–10 km) which are even finer than the internal Rossby radius. Since tracer concentration gradients are strong in the vertical, sub-mesoscale upwelling introduces an anomalous signature in the sea surface concentration of tracers, or can alternatively be thought of as introducing variance at the finest scales under consideration here. If the tracer considered is nutrients (nitrate or phosphate that are essential for phytoplankton production) then the upwelling results in the generation of small scale phytoplankton patches. This is the situation when the surface of the pelagic ocean is depleted of nutrients and phytoplankton production is limited by the supply of nutrients from the subsurface. Phytoplankton production takes up the nutrients from the surface, restoring it to its nutrient-depleted state. If the time scale for biological production in response to the nutrients is short, then the variance in phytoplankton is seen mostly at the small scales at which it was generated. If the response time scale is large, the nutrient patches generated by sub-mesoscale upwelling spread to larger regions due to the advection of the upwelling features in the flow. Consequently the variance is distributed to larger scales.

A balance between the processes generating small-scale variance (sub-mesoscale upwelling) and removing it (nutrient uptake) can be written as

$$w\frac{\partial c}{\partial z} \sim -\frac{c}{\tau} \tag{3}$$

where c is the anomalous tracer concentration at the surface (normalized by the mean), w is the upwelling velocity, z denotes the vertical coordinate direction, and τ is the response or removal time scale of the tracer (nutrient) in the surface layers. We scale this relation, by choosing W to represent the typical upwelling velocity, c and c_∞ to be the normalized concentration of the tracer at the surface and depth, and h to be the characteristic depth over which the upwelling occurs. Thus $w\partial c/\partial z \sim W(c - c_\infty)/h \sim -c/\tau$. Taking the logarithm of both sides (terms 2 and 3), using the definition of the patchiness index $p \equiv \log V/\log L$, and considering the variance spread over the same range of length scales for different tracers with different τ, we get the relation [*Mahadevan & Campbell* (2002)]

$$p \sim \log\bar{\tau} + \log c_\infty, \tag{4}$$

where $\bar{\tau}$ is the time scale τ normalized by the upwelling time scale h/W. The scaling suggests that the patchiness varies as the logarithm of the ratio of the time scales τ and h/W. The time scale h/W may be difficult to ascertain in the field but the spatial heterogeneity of various tracers can nonetheless be compared using this relation, because they are affected by the same flow field. Hence their patchiness p is expected to vary logarithmically with the characteristic time scale of the tracer's response to processes that modify it in the upper ocean. While this scaling relation is derived using a rather simplistic balance between two processes (sub-mesoscale upwelling and nutrient uptake), tracers in the ocean may be affected by a number of competing processes with several time scales. It is possible to account for multiple time scales by calculating an effective non-dimensional $\bar{\tau}$ using $\frac{1}{\bar{\tau}} = \frac{1}{\bar{\tau}_1} + \frac{1}{\bar{\tau}_2} + \frac{1}{\bar{\tau}_3}$....

This dependence of the spatial heterogeneity on the response time scale is tested with a three-dimensional fluid dynamical model of an ocean front that contains a tracer. The model is configured in an east-west periodic channel that is initialized with a north-south density gradient representative of an upper ocean front in the mid-latitudes. The front and the associated east-west jet are baroclinically unstable and form meanders and eddies, as are seen in the ocean. The tracer is initialized to resemble nitrate; it is absent from the surface and increases exponentially with depth. Any excursions of the tracer from this initial profile due to upwelling are restored to the initial state on a time scale τ, where τ is chosen to be 2.5,5,10,20 ,40 and 80 days. The spatial heterogeneity of the surface layer of the model is analyzed for the tracers with different values of τ at different instants in the flow using the variance based analysis (Figure 4). The slope of the V-L plot in log-log space gives the value of the patchiness index p. This index p is found to vary with the log of the time scale τ suggesting that spatial heterogeneity of tracers with different response time scales that are affected by the same fluid dynamical motions is related by the scaling relationship we derived.

The above discussed relationship between patchiness and response time applies to regions of the ocean where the phytoplankton growth is strongly limited by the upwelling of nutrients from the subsurface. Even though we neglected the effect of horizontal advection in generating small-scale variance while deriving the scaling relationship, the three-dimensional model results are fairly consistent. This is because the vertical motions make a dominant contribution to surface spatial distributions in the model, as is the case in many regions of the ocean. However, it is noteworthy that different results are obtained when the effects of horizontal advection are dominant as in the situation where phytoplankton blooms generate large scale gradients in Chl between bloom regions and clear waters. In this case, variance is introduced at the large scale and stirred in to smaller scale filaments by advection [*Martin* (2003)]. The relationship between the longevity of the tracer and its patchiness is just the opposite of what is derived above, because with time, the tracers develop finer scale structure. Abraham (1998) *Abraham* (1998) uses the balance between advection and reaction in a two-dimensional flow to explain the difference in patchiness between phytoplankton and zooplankton populations in a model. In his modeling study, the longer-lived zooplankton generate finer scales than the phytoplankton. This demonstrates how relevant processes are to the spatial heterogeneity.

Finally, we address the issue of modeling or observing different tracers given what we have learned about their spatial heterogeneity and its dependence on the response time τ. If our intention is to capture the bulk of the variance in the distribution of the tracer, then clearly, the tracer that is more patchy, requires higher spatial resolution for modeling or observing. If for example, we wish to capture 80% of the variance in a tracer distribution, then setting V, the non-dimensional variance as 0.8 in (2), and taking L to be the model resolution Δ, we get $\log \Delta \sim 1/p$, which when combined with (4)) gives

$$\Delta \sim \exp(-1/\log \bar{\tau}). \tag{5}$$

Thus two tracers that differ by a factor of 10 in their response times τ, differ by a factor of 4 in the resolution required to model or sample them. The faster acting tracer generates variance at finer scales and requires higher model and sampling resolution. Phytoplankton reproduction and organic matter remineralization time scales are typically on the order of a few days, and are shorter than the time scale of equilibration of physical properties in the oceanic mixed layer with the atmosphere. (For example, the time scale on which the surface ocean temperature equilibrates with the atmosphere may be approximately be a month, but the time scale for phytoplankton growth is only a few days.) This means that higher spatial resolution is needed to model and observe the phytoplankton ecology and carbon chemistry in the oceans, than the distribution of temperature. Further, the logarithmic dependence of the spatial heterogeneity on τ implies that models are much more sensitive to the choice of the process time scale τ, when τ is small.

Conclusions

By analyzing the distribution variance V of various properties in the surface ocean as a function of length scale L, and plotting $\log V$ *vs.* $\log L$, we find that $V \sim L^p$ for satellite derived fields

of sea surface Chl and temperature on length scales ranging from 1–500 km. We use the slope p of the V-L curves as a measure of the spatial heterogeneity or patchiness; smaller p corresponds to greater patchiness. We develop a relationship between the spatial heterogeneity of sea surface tracers and the time scale of processes that modify them by considering the balance between processes that generate variance at one scale and processes that shift the variance to another scale. The distributions of tracers at the sea surface are affected by sub-mesoscale upwelling which introduces variance at small scales, and also by processes like air-sea exchange, heat flux, evaporation or biological production, that remove this variance and restore the surface ocean to an ambient state. The ratio between the time scale of upwelling and the time scale of the process that modifies the tracer in the upper ocean τ, determines the patchiness of the system. For various tracers affected by the same flow field, the patchiness index $p \sim \log \bar{\tau}$. This relationship enables us to relate the spatial heterogeneity of different tracers with different response times. It also enables us to estimate the model resolution needed to model the variability of different substances in the ocean.

Finally, it is worth putting this oceanographic study in the context of terrestrial studies and asking whether our approach is generalizable to other systems. Distributions in the ocean tend to be transient and coupled to the dynamics of the fluid medium. Since properties in the ocean are evolving, we use a measure of spatial heterogeneity that is independent of the specifics of the distribution, but characterizes it in terms of the distribution of variance over length scales. Such measures of heterogeneity can also be used for terrestrial systems if the appropriate variable (such as the concentration of a substance or property) can be measured. Assuming a steady state in the variance distribution, we then balance the factors that change the variance in the system in opposing ways. This approach could be applied to systems where the processes generating or changing variance are understood and quantifyable, and when a steady state can be assumed in the variance characteristics. But many terrestrial systems evolve very slowly and hence it is difficult to measure the processes that modify them and their rates of modification, particularly when the evolution of the system occurs in a highly erratic fashion. Nonetheless, we hope that this study would provoke thinking about what properties and parameters need to be measured in a system to explain its heterogeneity. Further, various measures for spatial heterogeneity might be conceived for different systems. If indeed relationships can be found between these measures and the processes that alter the system, then it is conceivable that the evolution of the spatial heterogeneity will be predictable from information about the processes, or inversely, the spatial heterogeneity of a system will provide information about the rates or strengths of the processes at play.

Acknowledgments: The SeaWiFS and MODIS data were obtained from the NASA Goddard DAAC. This work was sponsored by ONR (N00014-00-C-0079) and NASA (NAS5-96063 and NAG5-11258). Thanks for Steve Groom and Timothy Moore for Figs. 1 & 2.

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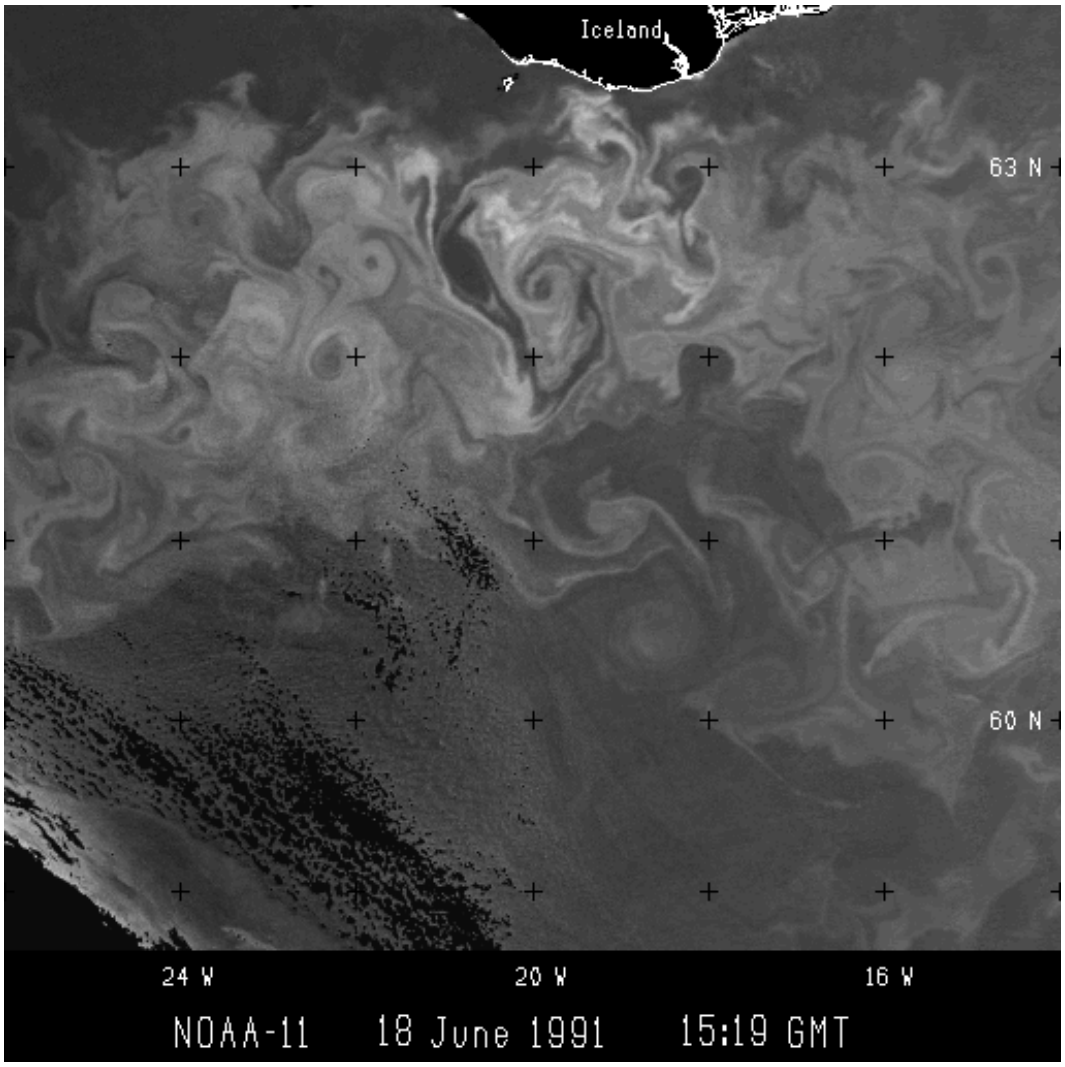
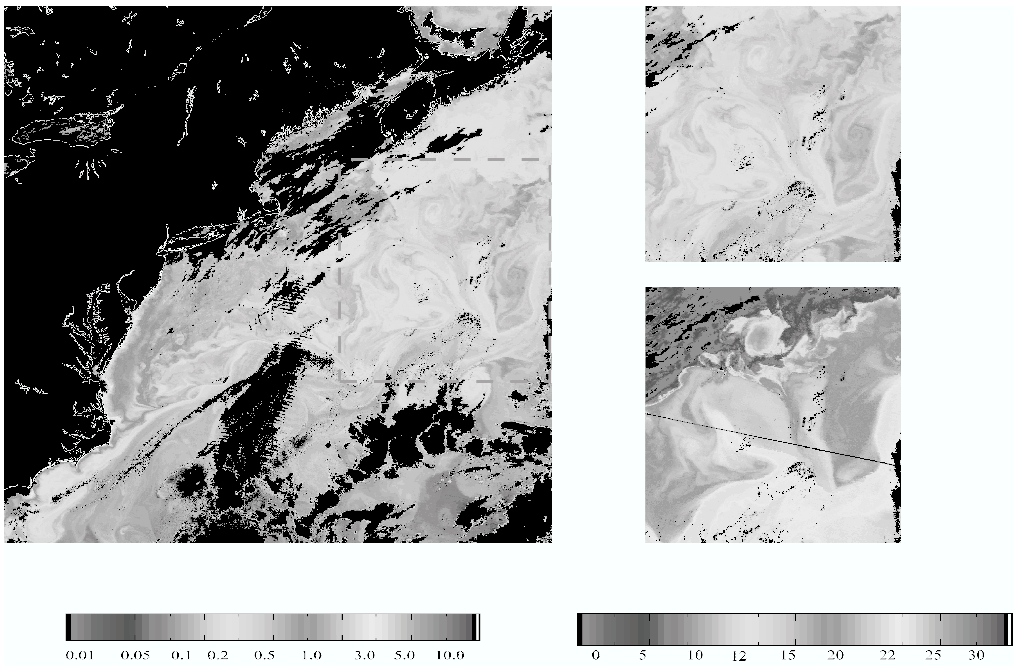


Figure 1: An example of spatial heterogeneity in the ocean is seen in this Advanced Very High Resolution Radiometer (AVHRR) satellite image of a coccolithophore bloom south of Iceland in June 1991. Coccolithophores are a kind of phytoplankton that grows calcium carbonate plates that are shed, making the water appear milky in these images. The small crosses in the figure are 110 km apart. The image shows the strong coupling between physical flow fields and the biological distributions in the upper ocean. (AVHRR image received at the NERC Receiving Station Dundee and processed at the Plymouth Marine Laboratory – courtesy Steve Broom.)



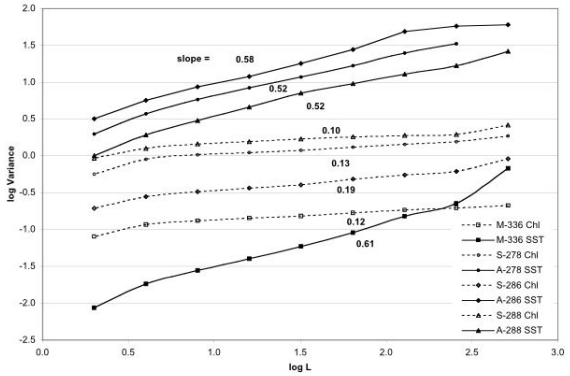


Figure 2: (a) Satellite image of sea surface chlorophyll (Chl) in the Atlantic Ocean acquired by the Moderate Resolution Imaging Spectroradiometer (MODIS). The boxes on the right show simultaneous views of the SST and Chl for the dashed region in the larger picture. This region is 512×512 km^2.The key is logarithmic for Chl, but linear for SST. (b) The variance *vs.* length-scale, i.e. V-L, relationship for Chl and SST plotted on $\log - \log$ axes for simultaneous satellite data of SST (bold) and Chl (dotted) as that shown above. The M-336 curves are based on MODIS data from the Arabian Sea. In addition, we show results for three concurrent AVHRR (A-278, A-286, and A-288) SST and SeaWiFS (S-278, S-286, and S-288) Chl images from the N. Atlantic in October 2000. The length scales analyzed range from 2 to 512 km. The slopes indicated are estimated for lines fitted to the points between $L = 4$ km and 256 km. (Published in *Mahadevan and Campbell, 2002.*)

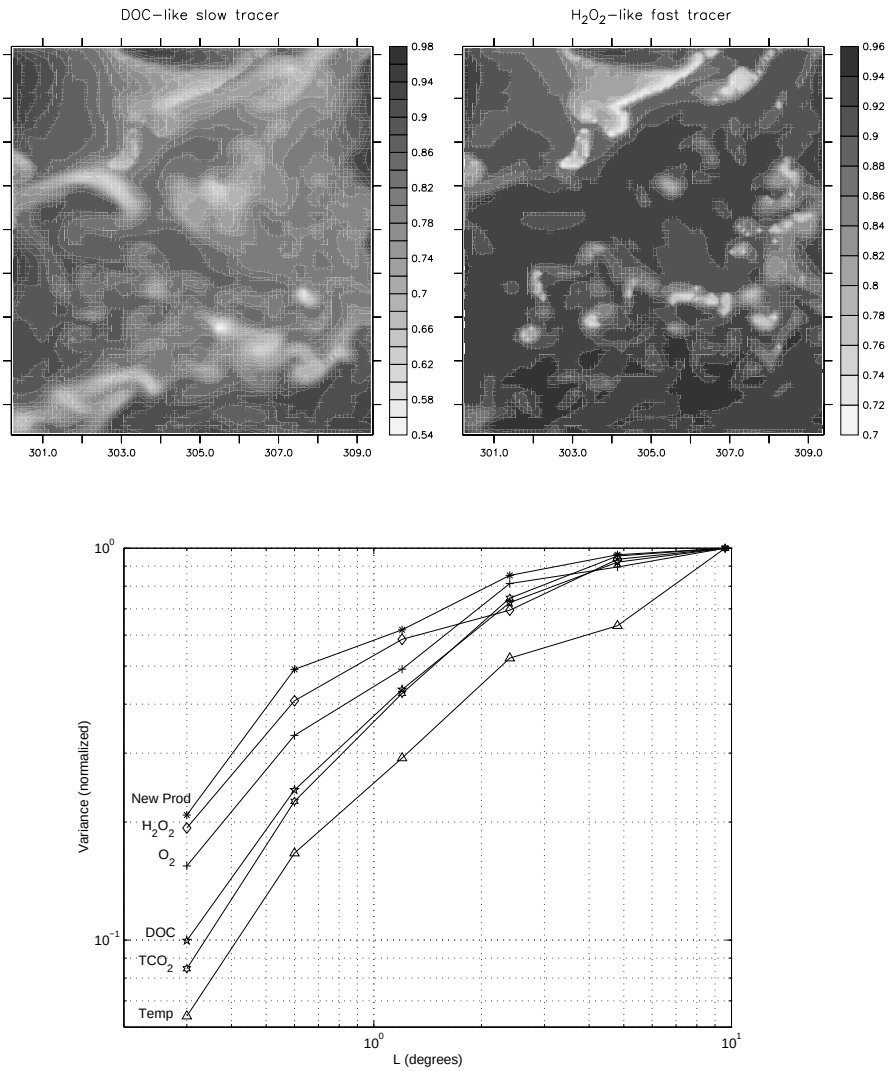


Figure 3: (a) Surface distributions of a fast and slowly responding tracer within a model of a $10^0 \times 10^0$ region of the Atlantic. The model was driven by time-dependent boundary conditions extracted from a global circulation model and run at a resolution of 0.1^0 latitude-longitude. The tracers shown are the fast-acting H_2O_2-like tracer and the slower DOC-like tracer. They were initialized as being 1 at the surface and decreasing exponentially to 0 over an e-folding depth of 100 m. When the tracers deviated from this distribution due to advection, they were restored to it with an e-folding time of 3 and 60 days, respectively. The tracer with the shorter response time is seen to develop finer scale structure. (b) The $\log V$ *vs.* $\log L$ curves demonstrate that the different tracers used in this model (temperature, dissolved inorganic carbon (DIC), oxygen (O_2), new production and the H_2O_2-like fast and DOC-like slow tracers) develop different spatial heterogeneities. The faster acting tracers are more patchy and have smaller p than the slower acting ones. (Published in *Mahadevan and Campbell, 2002.*)

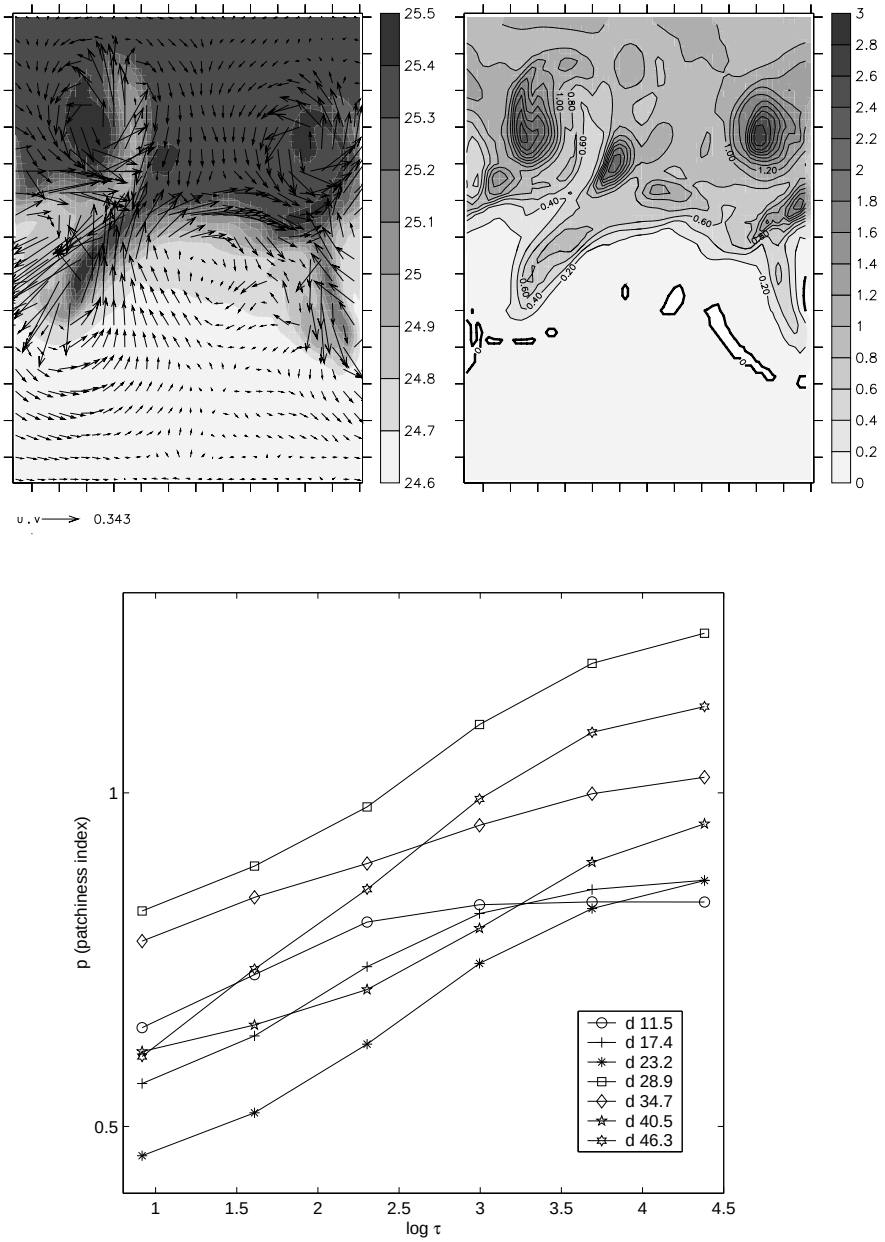


Figure 4: Top left: Surface density field and flow vectors in a model simulation of a baroclinically unstable evolving front shown 17 days after initialization. The domain size is 258 km×285 km and the horizontal resolution is approximately 4 km. Top right: Sureface view of the tracer (nutrient) distribution in the upper 95 m. The tracer shown has a response time τ of 40 days. Tracers with a lessor value of τ show a similar distribution but are more patchy and less abundant. Below: The slope of the $\log V$ *vs.* $\log L$ curves for the tracers in the model plotted *vs.* the value of τ (on a log axis) to verify that $V \sim \log \tau$. Each curve corresponds to a different time in the simulation at which the spatial heterogeneity was analyzed. (Previously published in *Mahadevan and Campbell, 2002.*)